Fragmentation Functions
measurements at Belle

DIS 2018, Kobe,
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Ralf Seidl (RIKEN)
Outline

- Single hadron fragmentation
  - Hyperon and charmed Baryon fragmentation
  - $\Lambda$ polarizing fragmentation
- Di-hadron fragmentation
  - Unpolarized mass, $z$ dependence
- Other ongoing measurements ($kt$ dependence)
**Access to FFs**

- **SIDIS:**
  \[ \sigma^h(x, z, Q^2, P_{h\perp}) \propto \sum_q e^2_q q(x, p_t, Q^2) D^h_{1,q}(z, k_t, Q^2) \]
  - Relies on unpol PDFs
  - Parton momentum known at LO
  - Flavor structure directly accessible
  - Transverse momenta convoluted between FF and PDF

- **pp:**
  \[ \sigma^h(P_T) \propto \int \sum_{x_1, x_2, z} f_a(x_1) \otimes f_{a'}(x_2) \otimes \sigma_{aa'} \otimes D^h_{1,q}(z) \]
  - Relies on unpol PDFs
  - leading access to gluon FF
  - Parton momenta not directly known

- **e+e-:**
  \[ \sigma^h(z, Q^2, k_t) \propto \sum_q e^2_q \left(D^h_{1,q}(z, k_t, Q^2) + D^h_{1,q}(z, k_t, Q^2)\right) \]
  - No PDFs necessary
  - Clean initial state, parton momentum known at LO
  - Flavor structure not directly accessible
Belle Detector and KEKB

- Asymmetric collider
- $8\text{GeV} \ e^- + 3.5\text{GeV} \ e^+$
- $\sqrt{s} = 10.58\text{GeV} \ (\Upsilon(4S))$
- $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B \ \bar{B}$
- Continuum production: $10.52\text{ GeV}$
- $e^+e^- \rightarrow q \ q \ (u,d,s,c)$
- Integrated Luminosity: $>1000 \text{ fb}^{-1}$
- $>70\text{fb}^{-1} \Rightarrow \text{continuum}$

Good tracking and particle identification!

- $\varepsilon(K) \sim 85\%$
- $\varepsilon(\pi \rightarrow K) < 10\%$

- Integrated Luminosity
- Central Drift Chamber
- Small cell +He/C$_2$H$_6$
- Aerogel Cherenkov counter
- n=1.015~1.030
- SC solenoid
- 1.5T
- CsI(TI)
- $16X_0$
- TOF counter
- Good tracking and particle identification!
Single hadron fragmentation

\[ D_{1,q}^h (z, Q^2) \]

In $e^+e^-$ annihilation:

\[ Q = \sqrt{s} \]

\[ z = \frac{2E_h}{Q} \approx \frac{E_h}{E_q} \]
Hyperons similar to light hadron fragmentation → peaking at low $z$ ($x_p$)

Baryon production not too well described by Pythia 6 default settings
Charmed baryons carry large fraction of parton momentum, similar to charmed mesons
Charmed fragmentation reasonably described in Pythia for main states
Baryon production rates

- First feed-down corrected production rates extracted
- No $\Lambda(1520)$ enhancement seen
- Strangeness suppression seen for hyperons:
  \[
  \frac{\sigma(S = -1)}{2J + 1} > \frac{\sigma(S = -2, -3)}{2J + 1}
  \]
- Difference in slopes for $\Lambda_c$ and $\Sigma_c$ in support of diquark production picture (spin 1 diquarks suppressed)
Single $\Lambda$ polarization measurements

- Related to open question about $\Lambda$ polarization in hadron collisions from 40 years ago!
- Fragmentation counterpart to the Sivers Function: unpolarized parton fragments into transversely polarized baryon with transverse momentum wrt to parton direction
- Reconstruct $\Lambda$, its transverse momentum and polarization

YingHui Guan (Indiana/KEK): arXiv:1611.06648

$$D_{1,q}^{\perp h}\left(z, k_T, Q^2\right)$$
Transverse momentum dependence

- Different behavior for low and high-z:
  - At low z small
  - At intermediate z falling polarization with $k_t$
  - At high z increasing polarization with $k_t$
Interesting $z_\pi$ and $z_\Lambda$ dependence:

- At low $z_\Lambda$ light quark fragmentation dominant, some charm in $\pi^-$ $\rightarrow$ different signs
- At high $z_\Lambda$ strange + charm fragmentation more relevant $\rightarrow$ same signs
Di-hadron fragmentation functions

\[ D_{1,q}^{h_1,h_2} (z, m, Q^2) \]

\[ H_{1,q}^{h_1,h_2} (z, Q^2, M_h) \]

\[ D_{1,q}^{h} (z_1, Q^2) D_{1,q}^{h} (z_2, Q^2) \]
Di-hadrons

- Single inclusive hadron multiplicities ($e^+e^- \to hX$) sum over all available flavors and quarks and antiquarks:
  \[
  \frac{d\sigma(e^+e^- \to hX)}{dz} \propto \sum_q e_q^2 (D_{1,q}^h(z, Q^2) + D_{1,\bar{q}}^h(z, Q^2))
  \]

- Especially distinction between favored (ie $u \to \pi^+$) and disfavored ($\bar{u} \to \pi^+$) fragmentation would be important

- Idea: Use di-hadron fragmentation, preferably from opposite hemispheres and access favored and disfavored combinations:
  \[
  u\bar{u} \to \pi^+\pi^- X \propto D_{u,fav}^\pi(z_1, Q^2) \cdot D_{u,fav}^\pi(z_2, Q^2) + D_{u,dis}^\pi(z_1, Q^2) \cdot D_{u,dis}^\pi(z_2, Q^2)
  \]
  \[
  u\bar{u} \to \pi^+\pi^+ X \propto D_{u,fav}^\pi(z_1, Q^2) \cdot D_{u,dis}^\pi(z_2, Q^2) + D_{u,dis}^\pi(z_1, Q^2) \cdot D_{u,fav}^\pi(z_2, Q^2)
  \]

- Also: unpol baseline for interference fragmentation
Ratios to opposite charge pion pairs

\[ R \approx \frac{D_{\text{dis}}(z_1)D_{\text{fav}}(z_2) + D_{\text{fav}}(z_1)D_{\text{dis}}(z_2)}{D_{\text{fav}}(z_1)D_{\text{fav}}(z_2) + D_{\text{dis}}(z_1)D_{\text{dis}}(z_2)} \]

$\pi^+\pi^+$ comparable to $\pi^+\pi^-$ at low $z$, decreasing towards high $z$:

$\rightarrow$ Favored and disfavored fragmentation similar at low $z$

$\rightarrow$ Disfavored much smaller at high $z$
Hemisphere composition

Same hemisphere contribution drops rapidly: Consistent with LO assumption of Same hemisphere: single quark $\rightarrow$ di-hadron FF: $(z_1 + z_2 < 1)$

Opposite hemisphere: single quark $\rightarrow$ single hadron FF

Diagonal $z_1, z_2$ bins

Belle: RS et al., PRD92 (2015) 092007

Systematic uncertainties not displayed
Explicit di-hadron mass dependence

- Global fits currently missing unpolarized di-hadron FF baseline

→ Belle to the rescue
- Use same hemisphere di-hadrons for this analysis
- 16 z bins between 0.2 – 1
- 100 mass bins between 0.3 – 2.3 GeV
- Data analysis and correction steps same as previous di-hadron analysis, except for ISR treatment
Di-hadron mass dependence

Similar analysis in same hemisphere and mass – combined z binning. Important input for IFF based transversity global analysis

Belle: RS et.al. PRD96 (2017), 032005
Mass dependence comparisons to Pythia tunes

Magnitude and z dependence reasonable in Pythia 6.4 default, Intermediate mass structure better described by LEP tunes (higher spin mesons)
Di-pion individual contributions
Contributions from various resonances and direct fragmentation

Belle: RS et.al. PRD96 (2017), 032005

\[ \frac{d^2\sigma}{dz dm_{\pi\pi}} \]
Transverse momentum dependence

Aka un-integrated PDFs and FFs

\[ D_{1,q}^h(z, Q^2, k_t) \]
**K_T Dependence of FFs in e+e-**

- Gain also sensitivity into transverse momentum generated in fragmentation

- Two ways to obtain transverse momentum dependence
  - Traditional 2-hadron FF
    - Use transverse momentum between two hadrons (in opposite hemispheres)
    - Usual convolution of two transverse momenta
  - Single-hadron FF wrt to Thrust or jet axis
    - No convolution
    - Need correction for q\̅q axis
MC sample for various hadrons

\[ \frac{d^2 \sigma}{dzdk_T} \]

**MC simulation**

- π⁺
- K⁺
- p⁺
Summary and outlook

- Hyperon and charmed baryon fragmentation measurements just published, support for diquark picture in charm FF
- Nonzero Lambda polarization measured, interesting flavor dependence
- Di-hadron fragmentation functions measured, important input for di-hadron related Transversity/Tensor charge extractions
- Transverse momentum dependent fragmentation analysis ongoing
- Other results being finalized as well ($\eta, \pi^0$ Collins)
Full results for pion pairs

PRD92 (2015) 092007

Pion pair example in any topology combination shown here

\[ \frac{d^2\sigma}{dz_1 dz_2} \]

\( z_2 \)
Low $z$ dominates integral: 
→ Well defined, all tunes agree

High $z$ not well measured, especially at Belle energies: 
→ large spread in tunes

Default Pythia settings and current Belle setting with good agreement

Results for diagonal $z_1 z_2$ bins

PRD92 (2015) 092007
Di-hadron mass dependence

Pion – kaon pairs

Pion-kaon individual contributions


\[
d^2\sigma \over dz dm_{\pi K}
\]

4/18/2018

R.Seidl: Belle Fragmentation
Kaon pairs

Kaon-kaon individual contributions

Belle: RS et.al. [1706.08348]

\[ \frac{d^2\sigma}{dzdm_{\pi K}} \]

\[ m_{\pi K} \]

4/18/2018

R. Seidl: Belle Fragmentation
Differences in Pythia/JetSet settings

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VM suppression
$P_x, P_y$ Gauss width
Lund params
$\Lambda_{QCD}$ and $E$ cutoff
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<th>Parameter</th>
<th>Description</th>
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<td>PARJ(1)</td>
<td>Diquark suppression relative to quark antiquark production</td>
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<td>PARJ(2)</td>
<td>Strangeness suppression relative to u or d pair production</td>
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<td>PARJ(3)</td>
<td>Extra suppression of strange diquarks relative to strange quark production</td>
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<td>Axial ((ud_1)) vs scalar ((ud_0)) diquark suppression</td>
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<td>PARJ(11)</td>
<td>Light meson with spin 1 probability</td>
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<td>PARJ(12)</td>
<td>Strange meson with spin 1 probability</td>
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<td>PARJ(13)</td>
<td>Charm meson with spin 1 probability</td>
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<td>PARJ(14)</td>
<td>Spin 0 meson with (L = 1) and (J = 1) probability</td>
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<td>PARJ(15)</td>
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<td>PARJ(16)</td>
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<td>Spin 1 meson with (L = 1) and (J = 2) probability</td>
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<td>PARJ(19)</td>
<td>Extra baryon suppression relative to regular diquark suppression (if MSTJ(12) = 3)</td>
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<td>PARJ(21)</td>
<td>Gaussian Width of (p_x) and (p_y) for primary hadrons</td>
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<td>PARJ(25)</td>
<td>(\eta) production suppression factor</td>
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<td>(\eta') production suppression factor</td>
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<td>PARJ(42)</td>
<td>Lund b parameter: (exp(-b m_1^2/z))</td>
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<td>PARJ(45)</td>
<td>Addition to a parameter for diquarks</td>
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<td>PARJ(46)</td>
<td>Modification of Lund fragmentation for heavy quarks with Bowler, charm, bottom</td>
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<td>PARJ(54)</td>
<td>Charm fragmentation functional form and value if MSTJ(11) = 2 or 3</td>
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<td>Bottom fragmentation functional form and value if MSTJ(11) = 2 or 3</td>
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<td>PARJ(81)</td>
<td>(\Lambda_{QCD}) for parton showers</td>
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<td>PARJ(92)</td>
<td>Invariant mass cut-off for parton showers</td>
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Di-hadron fragmentation functions

\[ D_{1,q}^{h_1 h_2}(z, m, Q^2) \]

\[ D_{1,q}^{h}(z_1, Q^2) D_{1,q}^{h}(z_2, Q^2) \]
Setup

- Generally look at $4 \times 4$ hadron combinations ($\pi$, $K$, +,-)
  - Keep separate until end: only 6 independent yields
- 3 hemisphere combinations:
  - same hemisphere (thrust $>0.8$)
  - opposite hemisphere (thrust $>0.8$)
  - any combination (no thrust selection)
- $16 \times 16$ $z_1 z_2$ binning between $0.2 - 1$
## Correction chain

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<th>Correction</th>
<th>Method</th>
<th>Systematics</th>
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<td>PID mis-id</td>
<td>PID matrices (5x5 for $\cos \theta_{\text{lab}}$ and $p_{\text{lab}}$)</td>
<td>MC sampling of inverted matrix element uncertainties, variation of PID correction method</td>
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<td>Momentum smearing</td>
<td>MC based smearing matrices (1600x1600), SVD unfold</td>
<td>SVD unfolding vs analytically inverted matrix, reorganized binning, MC statistics</td>
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<td>Non-qqbar BG removal</td>
<td>eeeu, eess, eecc, tau MC subtraction</td>
<td>Variation of size, MC statistics</td>
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<td>Acceptance I (cut efficiency)</td>
<td>In barrel reconstructed vs udsc generated in barrel</td>
<td>MC statistics</td>
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<td>Acceptance II</td>
<td>udsc Gen MC barrel to $4\pi$</td>
<td>MC statistics, variation in tunes</td>
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<td>Weak decay removal (optional)</td>
<td>udcs check evt record for weak decays</td>
<td>Compare to other Pythia settings</td>
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<td>ISR</td>
<td>ISR on vs ISR off in Pythia</td>
<td>Variations in tunes</td>
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R.Seidl: Belle Fragmentation
Using Martin Leitgab’s 5x5 PID matrices in fine 17 x 9 \( P_{\text{lab}} \times \cos \theta_{\text{lab}} \) binning for both hadrons:
- confirmed unitarity
- confirmed \( \pi \pi, \pi K, \) KK yields by comparing to D\(^0\) BRs.
- Reduced smearing matrices from 1600 x 1600 to filled (ie kinematically reachable bins)
- Using SVDUnfold Method in Root, generally good statistics and little regularization necessary
Non-qqbar removal:

Remove all two-photon and tau events from yields, contributions generally up to several %, slightly higher for kaon related pairs.
Stacked, relative contributions
Acceptance correction

ACCI: Reconstruction and efficiency correction in Barrel acceptance

ACCII: Barrel to $4\pi$ correction

$K_S$ drop in efficiency due to SVD 3 hit requirement - increasing effect with $z$ due to boost
Weak correction (optional)

Traced in gen MC hadrons back to mothers with non ud content $\rightarrow$ if not vetoed (K*, ssbar, ccbar resonances, some hyperons and excited states) $\rightarrow$ Weak

Currently weak decays (based on MC) not removed: both values provided in paper.
ISR correction

All different tunes very similar except old Belle tune \(\rightarrow\) assigned as systematics
-high mass drop of ratio due to boost
Overall systematic uncertainties

Systematic uncertainties dominated by acceptance correction (for different tunes), PID uncertainties and ISR correction
Unpolarized fragmentation functions

\[ D_{1,q}^h(z, Q^2) \]
New addition: single protons

PRD92 (2015) 092007

- Default Pythia and current Belle in good agreement with pions and kaons
- Protons not well described by any tune
MC sample for various hadrons

MC simulation

\[ \frac{d^2\sigma}{dzdk_T} \]

\[ k_T \]

\[ \pi^+ \]

\[ K^+ \]

\[ p^+ \]
MC examples vs $k_T^2$

Fit exponential to smaller transverse momenta for Gaussian $k_T$ dependence and power low at higher $k_T$

MC simulation
MC Gaussian widths

Once available for data this will be the first direct (no convolutions) measurement of z dependence of Gaussian widths.

\[ \sigma_{\pi^\pm} \quad \sigma_{p/p} \]

MC simulation
Charmed Fragmentation

Heavier particles generally plotted vs normalized momentum $x_p = \frac{p^h}{p_{max}^h}$

Unlike light hadrons charmed hadrons contain large fraction of charm quark momentum

PRL.95, 142003 (2005) (Babar)
PRD73, 032002 (2006) (Belle)
PRD75, 012003 (2007) (Babar)
PRL 99, 062001 (2007) (Babar)
Collins fragmentation function


\[ D_{q}^{h}(z, P_{h\perp}) = D_{1,q}^{h}(z, P_{h\perp}^{2}) + H_{1,q}^{h}(z, P_{h\perp}^{2}) \left( \frac{\hat{k} \times \mathbf{P}_{h\perp}}{zM_{h}} \right) \cdot \mathbf{S}_{q} \]

- Spin of quark correlates with hadron transverse momentum
- Translates into azimuthal anisotropy of final state hadrons
Belle Collins asymmetries

- **Red points**: \( \cos(\phi_1 + \phi_2) \) moment of *Unlike* sign pion pairs over *like* sign pion pair ratio: \( A_{UL} \)
- **Green points**: \( \cos(\phi_1 + \phi_2) \) moment of *Unlike* sign pion pairs over any charged pion pair ratio: \( A_{UC} \)
- Collins fragmentation is large effect
- Consistent with SIDIS indication of sign change between favored and disfavored Collins FF

RS et. Al. (Belle), PRL96: 232002
PRD 78:032011, Erratum D86:039905
Interference Fragmentation (IFF) in $e^+e^-$

- $e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet}_1}(\pi^+\pi^-)_{\text{jet}_2}X$


- Early work by Collins, Heppelmann, Ladinsky [NPB 420 (1994)]

- Model predictions by:
  - Jaffe et al. [PRL 80, (1998)]
  - Radici et al. [PRD 65, (2002)]

$$A \propto H^\perp(z_1, m_1)H^\perp(z_2, m_2) \cos(\varphi_1 + \varphi_2)$$
Belle IFF asymmetries: \((z_1 \times z_2)\) Binning

Magnitude increasing with \(z\)

PRL107:072004(2011)
Belle IFF asymmetries: \((z_1 \times m_1)\) Binning

2 d distributions of one hemisphere

PRL107:072004(2011)
What are fragmentation functions?

How do quasi-free partons fragment into confined hadrons?

- Does spin play a role? Flavor dependence?
- What about transverse momentum (and its Evolution)?

What experiments measure:

- Normalized hadron momentum in CMS: \( e^+e^- \rightarrow h(z) X ; z = \frac{2E_h}{\sqrt{s}} \)
- Hadron pairs’ azimuthal distributions: \( e^+e^- \rightarrow h_1 h_2 X ; \langle \cos(\phi_1+\phi_2) \rangle \); Collins FF, Interference (IFF)
- Cross sections or multiplicities differential in z: ep->hX, pp->hX

Additional benefits of the FF measurements:

- Pol FFs necessary input to transverse spin SIDIS und pp measurements to extract Transversity distributions function
- Flavor separation of all Parton distribution functions (PDFs) via FFs (including unpolarized PDFs)
- Baseline for any Heavy Ion measurement
- Access to exotics?
Unpolarized fragmentation functions:
- Provide flavor information in nucleon
- Most apparent in SIDIS measurements related to $\Delta q(x)$
- But also required for all RHIC hadron asymmetries (especially pion $A_{LL}$ charge ordering)
- Transverse momentum dependence needed for Sivers and other TMDs

Polarized fragmentation functions:
- For transverse spin almost unique access (require two chiral-odd functions):
  - DY: $\delta q \times \delta q$ or
  - SIDIS/RHIC: $\delta q \times$ Collins or $\delta q \times$ IFF
- FFs from Belle/Babar