Study of $K^0_S$ pair production in single-tag two-photon collisions at Belle

Sadaharu Uehara (KEK)
Belle Collaboration

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Apr. 16-20, 2018
Two-photon Physics at $e^+e^-$ collider

$\gamma^*(\gamma^*) \rightarrow \text{hadron(s)}$ (Exclusive final state):
- Useful to Test of QCD
- Measurement of resonance production and its properties
- Spectroscopy and new-resonance search

**Physics motivations of Single-tag measurements, $\gamma^*\gamma$:**

- $Q^2$ dependence of transition form factor (TFF) of resonances
  - $\rightarrow$ Test of QCD, models of meson/exotics, Hadron tomography by GDAs
- Reference of Light-by-Light hadronic contribution for $g-2|_{\mu}$
Measurement of single-tag processes and Form factor

**Reaction:** $e^+e^- \rightarrow e\,(e)\,\text{hadrons}$:

- (e) not detected going extremely forward

$\gamma\gamma$ cross section $\sigma(W, Q^2)$ is derived using Equivalent Photon Approximation (luminosity function).

$W$ -- $\gamma\gamma$ c.m. energy, $Q^2 = -q_1^2$ virtuality

In identical neutral meson pair production (C-even), the Bremsstrahlung diagram (C-odd) is not mixed.

**Transition form factor (TFF) of a resonance:** $F(Q^2)$

Proportional to the helicity amplitude of the resonance production

$$\sum \lambda \left| F(Q^2) \right|_\lambda^2 \propto \sigma (\gamma\gamma \rightarrow \text{Resonance})$$
**KEKB Accelerator and Belle Detector**

- **Asymmetric e⁻ e⁺ collider**
  
  8 GeV e⁻ (HER) x 3.5 GeV e⁺ (LER)
  
  $\sqrt{s} = \text{around 10.58 GeV} \Leftrightarrow \Upsilon(4S)$
  
  Beam crossing angle: 22 mrad

- **World-highest Luminosity**
  
  $L_{\text{max}} = 2.1 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1}$
  
  $\int L \, dt \sim 1040 \text{ fb}^{-1}$ (Completed in Jun.2010)

High momentum/energy resolutions

- CDC+Solenoid, CsI
- Vertex measurement – Si strips
- Particle identification
  
  TOF, Aerogel, CDC-dE/dx,
  
  RPC for $K_L$/muon

*S. Uehara, KEK, DIS2018, Apr.2018*
γ*γ \rightarrow \pi^0\pi^0: f_0(980) and f_2(1270) TFF’s

No-tag
(Q^2=0)

Single-tag
(Q^2=3-30 GeV^2)

The f_0/f_2 ratio larger than in the no-tag case.

Different Q^2 dependences in the helicities.

Theoretical predictions:
- Schuler, Berends, van Gulik, a heavy quark approx. NPB 523, 423 (1998) (SBG)
- Pascalutes, Pauk, Vanderhaeghen, saturated sum rule, PRD 85, 116001 (2012), η’s
- ibid., axial-vector mesons
How about in the $K^0_S K^0_S$ process?

Maximum at the $f_2'(1525)$ peak

$f_2(1270)/a_2(1320)$ destructive interference

Two-photon coupling of $f_0(1710)$

No data near the $K^0_S K^0_S$ mass threshold

lack of trigger efficiency for low-$p_t$ tracks

<table>
<thead>
<tr>
<th>Interference</th>
<th>$N_{\chi_{c0}}$</th>
<th>$N_{\chi_{c2}}$</th>
<th>$-2\ln \mathcal{L}/\text{ndf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>not included</td>
<td>248.3$^{+17.9}_{-17.2}$</td>
<td>53.0$^{+8.4}_{-7.4}$</td>
<td>57.34/73</td>
</tr>
<tr>
<td>included</td>
<td>266$^{+53}_{-53}$</td>
<td>53$^{+14}_{-12}$</td>
<td>57.22/71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interference</th>
<th>$\Gamma_\gamma \mathcal{B}(\chi_{c0})$ (eV)</th>
<th>$\Gamma_\gamma \mathcal{B}(\chi_{c2})$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>not included</td>
<td>8.09$^{+0.58}<em>{-0.58}$ $^{+0.041}</em>{-0.041}$ $^{+0.037}<em>{-0.037}$ $^{+0.028}</em>{-0.028}$</td>
<td>0.27$^{+0.07}<em>{-0.07}$ $^{+0.03}</em>{-0.03}$</td>
</tr>
<tr>
<td>included</td>
<td>8.7$^{+1.7}<em>{-1.7}$ $^{+0.9}</em>{-0.9}$</td>
<td>0.27$^{+0.07}<em>{-0.07}$ $^{+0.03}</em>{-0.03}$</td>
</tr>
<tr>
<td>Belle 2007</td>
<td>7.0$^{+0.65}<em>{-0.65}$ $^{+0.31}</em>{-0.31}$</td>
<td>0.31$^{+0.05}<em>{-0.05}$ $^{+0.03}</em>{-0.03}$</td>
</tr>
<tr>
<td>PDG 2012</td>
<td>7.3$^{+0.5}_{-0.5}$</td>
<td>0.297$^{+0.026}_{-0.026}$</td>
</tr>
</tbody>
</table>
Experimental analysis of Single-tag $K^0_S K^0_S$

$e^+e^- \rightarrow e^-(e)^0_K K^0_S, \ K^0_S \rightarrow \pi^+ \pi^-$ \hspace{1cm} 759 fb$^{-1}$

**Topology:** 1 electron (or positron) and 4 charged pions

**Event Selection Criteria:**

- **for tracks** 5 tracks satisfy $p_t > 0.1 \text{ GeV/c}$, >=2 of them satisfy $p_t > 0.4 \text{ GeV/c}$, 1 of them satisfies e-identification and $p > 1.0 \text{ GeV/c}$
- **for $K^0_S$s** Charged $\pi/K$ separation
  - Reconstructed $K^0_S K^0_S$ masses (two-dimensional cut):
    - $492.6 < \text{ave}[M(K^0_S)s] < 502.6 \text{ MeV/c}^2$ and $\text{diff}[M(K^0_S)s] < 10\text{MeV/c}^2$
  - $K^0_S$ decay vertex: $0.3 < v_r < 8 \text{ cm}$
    - (a finite decay flight length in the $r\phi$ plane)

**Kinematical cuts** (Energy/momentum conservation and transverse-momentum balance)

$E_{\text{ratio}} = \frac{E_{\text{measured}}}{E_{\text{expected}}^{K^0_S K^0_S}}$ and $|\Sigma p_t^*|$

satisfy

$\sqrt{ \left( \frac{E_{\text{ratio}}-1}{0.04} \right)^2 + \left( \frac{|\Sigma p_t^*|}{0.1 \text{ GeV/c}} \right)^2 } \leq 1$
Reconstructed mass, angles and Energy of the Signal candidates

Reconstructed $K^0_S$ mass (with a looser cut)

Tag-$e$ angle  
$K^0_S$ angle  
$K^0_S$ energy (in lab. frame, for the signal-candidate events)

Uniform c.m. angular distribution for $K^0_S$ assumed in the signal MC

S. Uehara, KEK, DIS2018, Apr. 2018
Background processes

Rejection of non-exclusive background, $K^0_S K^0_S X$ using $|\Sigma p_t^*|$ vs. $E_{ratio}$

Control region    Signal region

14% background only for $W<1.3$ GeV

Systematic uncertainty

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking</td>
<td>2</td>
</tr>
<tr>
<td>Electron-ID</td>
<td>1</td>
</tr>
<tr>
<td>Pion-ID (for four pions)</td>
<td>2</td>
</tr>
<tr>
<td>$K^0_S$ reconstruction (for two $K^0_S$'s)</td>
<td>3</td>
</tr>
<tr>
<td>Kinematic selection</td>
<td>4</td>
</tr>
<tr>
<td>Geometrical acceptance</td>
<td>1</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>1 - 3</td>
</tr>
<tr>
<td>Background effect for the efficiency</td>
<td>2</td>
</tr>
<tr>
<td>Angular dependence of DCS</td>
<td>6 - 22</td>
</tr>
<tr>
<td>Background subtraction</td>
<td>3 - 7</td>
</tr>
<tr>
<td>No unfolding applied</td>
<td>1</td>
</tr>
<tr>
<td>Radiative correction</td>
<td>3</td>
</tr>
<tr>
<td>Luminosity function</td>
<td>4</td>
</tr>
<tr>
<td>Integrated luminosity</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>13 - 24</strong></td>
</tr>
</tbody>
</table>

Total: 13% - 24%
W dependence and $\gamma^*\gamma$ cross section at $Q^2$ bins

Threshold enhancement (including backgrounds)

No $a_2(1320)/f_2(1270)$ seen
\( \chi_{cJ} \) charmonia

Assume that in total 7 events (3 events) peaking near the \( \chi_{c0} \) (\( \chi_{c2} \)) mass are purely from the charmonium (backgrounds are estimated <1 event in total):

\[
\frac{d\sigma_{ee}}{dQ^2} = 4\pi^2 \left( 1 + \frac{Q^2}{M_R^2} \right) \frac{(2J + 1) 2d^2 L_{\gamma^*\gamma}}{dWdQ^2} \Gamma_{\gamma^*\gamma}(Q^2) B(K_S^0 K_S^0) \tag{Definition of \( \Gamma_{\gamma^*\gamma} \)}
\]

The first measurement of \( \chi_{cJ} \) in the single-tag two-photon production

Solid curve: SBG with the charmonium-mass scale \( \leftarrow \) much favored
Dashed curve: With the \( \rho \)-mass scale (VDM like)
Partial Wave Analysis for TFF of $f'_2(1525)$

Applied for $W<1.8$ GeV. We take into account partial waves up to $J=2$. $J=1$ does not couple with $K^0_S K^0_S$ ($\rightarrow J^P = 0^+$ and $2^+$)

$$\frac{d\sigma(\gamma^*\gamma \rightarrow K^0_S K^0_S)}{d\Omega} = \sum_{n=0}^{2} t_n \cos(n\varphi^*) ,$$

$\gamma^*\gamma$ c.m. frame
$z^*$ axis // $\gamma^*$
$x^*z^*$ plane includes tag-e

Resonance amplitude for $f'_2$, etc.

$$A_R^J(W) = F_R(Q^2) \sqrt{1 + \frac{Q^2}{m_R^2}} \sqrt{\frac{8\pi(2J+1)m_R}{W}} \times \sqrt{\Gamma_{tot}(W) \Gamma_{\gamma\gamma}(W) B(K^0_S K^0_S)}$$

TFF of $f'_2$ for helicity $i = \lambda$

$$\sqrt{r_{ifp}} F_{f_{2p}} \quad (i = 0, 1, 2)$$

$r_{0fp} + r_{1fp} + r_{2fp} = 1$

$S, D_0$, etc. --- Partial-wave amplitudes
$\varepsilon_0, \varepsilon_1$ --- Spin-dependent flux factor ratios for the virtual photon
$Y^m_j$ --- Spherical harmonics
Formalism of PWA and parametrizations

Problems: Low statistics

Only 3 out of $S, D_0, D_1$ and $D_2$ are independent

Non-unique solution (multiple solutions for resonances)

→ Parametrization of the amplitudes with modelled $W$ and $Q^2$ dependences

$$S = A_{BW}e^{i\phi_{BW}} + B_Se^{i\phi_{BS}},$$
$$D_i = \sqrt{r_{ifa}(Q^2)(A_{f_2(1270)} - A_{a_2(1320)})}e^{i\phi_{faD_i}}$$
$$+ \sqrt{r_{ifp}(Q^2)}A_{f_2'(1525)}e^{i\phi_{fpD_i}} + B_{Di}e^{i\phi_{BD_i}},$$

$$A_{BW}(W) = \sqrt{\frac{8\pi m_S}{W}} \frac{f_S}{m^2_S - W^2 - im_S g_S} \times \frac{1}{(Q^2/m_0^2 + 1)^{ps}},$$

Nominal fit
$B_S = 0$

-Destructive interference between $f_2(1270)$ and $a_2(1320)$
- $r_i(Q^2)$ and TFF for $f_2(1270)$ and $a_2(1320)$ are the same;
  use the values obtained in single-tag $\pi^0\pi^0$

$$\beta = \sqrt{1 - 4m_{K^0}/W^2} \text{ is the } K^0_S \text{ velocity}$$

$$r_{0fp} : r_{1fp} : r_{2fp} = k_0 Q^2 : k_1 \sqrt{Q^2} : 1$$

Determine each component and the relative phase by a fit
Fit results in W dependence at $Q^2$ bins

Show indications of:
- Non-zero $D_0$ and $D_1$ components in the $f_2'$ (1525).
- $f_2(1270)/\alpha_2(1320)$ not visible
- An enhancement near the threshold (0.995 GeV).
Angular dependence and the PWA fit

Due to a lack of statistics, we use $Q^2$-integrated angular differential cross section derived with the following convention (MC generated isotropically)

$$d^2\sigma / d|\cos \theta^*| d|\varphi^*| \propto N_{\text{EXP}}(|\cos \theta^*|, |\varphi^*|) / N_{\text{MC}}(|\cos \theta^*|, |\varphi^*|)$$

- $Q^2$: integrated over the full range between 3 and 30 GeV$^2$
- W: 4 bins

We regard this as the angular dependence at $<Q^2> = 6.5$ GeV$^2$

Fit:
- Black: total
- Red: $t_0$
- Blue: $t_1 \cos \varphi^*$
- Magenta: $t_2 \cos 2\varphi^*$

The fit is applied to the two-dimensional angular-dependence data. Forward enhancement is from the helicity-0 component.
$f'_2(1525)$ TFF Result

Shorter error bars; statistical
Longer error bars; statistical and systematic
Shaded areas; overall systematic

Schuler, Berends, van Glick (SBG)

- helicity-0 and -2 -- agree well with SBG.
- helicity-1 -- slightly smaller, but not inconsistent.

Note: the $Q^2$ dependence of each helicity fraction is assumed as follows

\[ r_{0\,fp} : r_{1\,fp} : r_{2\,fp} = k_0 Q^2 : k_1 \sqrt{Q^2} : 1 \]

Fractions $k_0$ and $k_1$ are floated.
The Threshold Enhancement

No $Q^2=0$ measurement

Mass threshold – 1.05 GeV

1.05 – 1.10 GeV

1.10 – 1.15 GeV

SBG:

$J^P=0^+$ state with $M=0.98$ GeV/$c^2$

- Not inconsistent with SBG.
- The limited statistics currently preclude a conclusive interpretation.
Summary

• Cross section for $\gamma^*\gamma \rightarrow K_S^0 K_S^0$ has been measured for $2M(K_S^0) < W < 2.6$ GeV, $3$ GeV$^2 < Q^2 < 30$ GeV$^2$

• $Q^2$ dependence of $\Gamma_{\gamma^*\gamma}$ of $\chi_{c0}$ and $\chi_{c2}$ has been measured. Preferable to the charmonium mass scale.

• $Q^2$ dependence of $f_2'(1525)$-TFF has been measured.

• Signature of an enhancement near the $K_S^0 K_S^0$ mass threshold is observed.

The measured $Q^2$ dependences are not inconsistent to theoretical predictions.
Backup
History of integrated luminosity at Belle

History of integrated luminosity at Belle

Luminosity at B factories

<table>
<thead>
<tr>
<th>(fb⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
</tr>
<tr>
<td>1400</td>
</tr>
<tr>
<td>1200</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>600</td>
</tr>
<tr>
<td>400</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>The Belle experiment started</td>
</tr>
<tr>
<td>2001</td>
<td>CP violation in B mesons was verified and the KEKB accelerator achieved the world’s highest luminosity</td>
</tr>
<tr>
<td>2002</td>
<td>Anomalous CP violation in b → s was measured</td>
</tr>
<tr>
<td>2003</td>
<td>The B → Kπ decay was discovered</td>
</tr>
<tr>
<td>2004</td>
<td>The New particle X (3872) was discovered</td>
</tr>
<tr>
<td>2005</td>
<td>Direct violation of CP in B → Kπ was found. The B → ργ decay was discovered</td>
</tr>
<tr>
<td>2006</td>
<td>B → τν was observed</td>
</tr>
<tr>
<td>2007</td>
<td>D meson mixing was discovered. A new particle composed of 4 quarks Z (4430) + was discovered</td>
</tr>
<tr>
<td>2008</td>
<td>Dr. Makoto Kobayashi and Dr. Toshihide Maskawa were awarded the Nobel Prize in Physics</td>
</tr>
<tr>
<td>2010</td>
<td>The Belle experiment was completed</td>
</tr>
</tbody>
</table>
Formalism of PWA

\[ |F(Q^2)| = \sqrt{\frac{\sigma_R^\lambda(Q^2)}{\sigma_R^\lambda(0)(1+\frac{Q^2}{M^2})}} \]

TFF is defined for each resonance R produced with each helicity \( \lambda \)

To obtain the resonance amplitudes:
Perform PWA, parameterizing W dependence of the resonance and continuum components of each helicity amplitude, e.g.,

\[ M_{++} = S + D_0, \]
\[ S = B_5(W) + A_{f0}(W) \]
\[ D_0 = 4\pi [\overline{B_{D0}(W)} + A_{f2}(W)\sqrt{r_{20}}] Y_2^0 \]

etc.

Determine each component as well as the relative phase by a fit

++ etc. --- Helicity state of the incident photons
S, D_0 etc. --- Partial-wave amplitude in \( \pi^0\pi^0 \) scattering
B, A_f --- Background and f-resonance components.
\( \varepsilon_0, \varepsilon_1 \) --- A spin-dependent flux factor ratio for the virtual-photons
\[ \gamma \gamma \rightarrow \pi^0 \pi^0 : f_0(980) \text{ and } f_2(1270) \text{ TFF's} \]

The curves are PWA fit constructed by parameterized \( f_0(980) \) and \( f_2(1270) \) etc. (see the paper)

| \cos 0^* | dependence for 
\( Q^2 = 9 \text{ GeV}^2 \) and different 
\( W \) bins

Significant contributions 
from \( \text{hel.}=0 \) and \( 1 \) in contrast 
to the no-tag (\( Q^2=0 \)) case

S.Uehara, KEK, DIS2018, Apr.2018
Two-photon decay width of $f_0(980)$ and $a_0(980)$

Predictions for $f_0(980)$

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Gamma_{\gamma \gamma}$ [eV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$uubar,dubar$</td>
<td>1300 – 1800</td>
</tr>
<tr>
<td>$ssbar$</td>
<td>300 – 500</td>
</tr>
<tr>
<td>$KKbar$ molecule</td>
<td>200 – 600</td>
</tr>
<tr>
<td>Four-quark</td>
<td>270</td>
</tr>
</tbody>
</table>

S. Uehara, KEK, DIS2018, Apr.2018
$f_0(1710)$ formation in $K^0_S K^0_S$

$K^0_S K^0_S$ is confirmed in two-photon process.

Assuming a single resonance, $J = 0$ or 2?  $J = 0$ is much preferred.

$\chi^2/\text{ndf}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$f_0(1710)$ fit</th>
<th>$f_2(1710)$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fit-H</td>
<td>fit-L</td>
</tr>
<tr>
<td>$\chi^2/\text{ndf}$</td>
<td>694.2/585</td>
<td>701.6/585</td>
</tr>
<tr>
<td>Mass($f_J$) (MeV/c$^2$)</td>
<td>$1750^{+5+29}_{-6-18}$</td>
<td>$1749^{+5+31}_{-6-42}$</td>
</tr>
<tr>
<td>$\Gamma_{\text{tot}}(f_J)$ (MeV)</td>
<td>$138^{+12+96}_{-11-50}$</td>
<td>$145^{+11+31}_{-10-54}$</td>
</tr>
<tr>
<td>$\Gamma_{\gamma\gamma}B(K\bar{K})_{f_J}$ (eV)</td>
<td>$12^{+3+227}_{-2-8}$</td>
<td>$21^{+6+38}_{-4-26}$</td>
</tr>
</tbody>
</table>

Two solutions of interference
Fit Results for resonances in $K^0_S K^0_S$

$f_2(2200)-f_0(2500)$ is the best solution (in all the J= 0, 2, 4 combinations)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$f_2(2200)$</th>
<th>$f_0(2500)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (MeV/$c^2$)</td>
<td>$2243^{+7+3}_{-6-29}$</td>
<td>$2539 \pm 14^{+38}_{-14}$</td>
</tr>
<tr>
<td>$\Gamma_{tot}$ (MeV)</td>
<td>$145 \pm 12^{+27}_{-34}$</td>
<td>$274^{+77+126}_{-61-163}$</td>
</tr>
<tr>
<td>$\Gamma\gamma B(K\bar{K})$ (eV)</td>
<td>$3.2^{+0.5+1.3}_{-0.4-2.2}$</td>
<td>$40^{+9+17}_{-7-40}$</td>
</tr>
</tbody>
</table>

Significances
- 3.4σ for $f_2(2200)$ over $f_0(2200)$
- 4.3σ for $f_0(2500)$ over $f_2(2500)$

- There can be an only wide state around 2240 MeV.
- Narrow appearances in previous measurements may be due to an interference effect and/or statistical fluctuation.
- A high-mass state at 2.5 GeV may be the heaviest light-quark scalar meson so far found.
Searching for non-$K^0_S K^0_S$ background, looking for an enhancement near $v_r=0$ in a loosely selected sample.

No enhancement. <1 event background in the final candidates.

Cross sections from positron-tag red, electron-tag black. Consistent each other.
### Table VI: Fitted parameters of cross sections and the number of solutions obtained under the conditions noted below. In each category, only solutions assuming \( k_0 \neq 0 \) \( \cap \) \( k_1 \neq 0 \) are shown. Only the single solution that gives the minimum \( \chi^2 \) in category 3 is shown, while two viable solutions in categories 1 and 2 are shown.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditions</td>
<td>( A_{BW} \neq 0 ) ( \cap ) ( B_S = 0 )</td>
<td>( A_{BW} = 0 ) ( \cap ) ( B_S \neq 0 )</td>
<td>( A_{BW} = B_S = 0 )</td>
</tr>
<tr>
<td>Number of solutions</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( \chi^2/ndf )</td>
<td>152.4/150</td>
<td>159.8/150</td>
<td>154.9/151</td>
</tr>
<tr>
<td>( k_0 ) (GeV(^{-2}))</td>
<td>(0.30_{-0.14}^{+0.31})</td>
<td>(0.31_{-0.15}^{+0.34})</td>
<td>(0.31_{-0.15}^{+0.34})</td>
</tr>
<tr>
<td>( k_1 ) (GeV(^{-1}))</td>
<td>(0.27_{-0.14}^{+0.30})</td>
<td>(0.27_{-0.15}^{+0.44})</td>
<td>(0.29_{-0.15}^{+0.35})</td>
</tr>
<tr>
<td>( F_{f_2p}(0.0);(\times 10^{-2}) )</td>
<td>(24.1_{-2.5}^{+2.6})</td>
<td>(24.4_{-2.5}^{+2.7})</td>
<td>(24.3_{-2.5}^{+2.6})</td>
</tr>
<tr>
<td>( F_{f_2p}(4.0);(\times 10^{-2}) )</td>
<td>(13.4_{-2.3}^{+2.6})</td>
<td>(13.9_{-2.3}^{+2.4})</td>
<td>(14.3_{-2.3}^{+2.5})</td>
</tr>
<tr>
<td>( F_{f_2p}(6.0);(\times 10^{-2}) )</td>
<td>(11.2_{-2.2}^{+2.3})</td>
<td>(11.3_{-2.2}^{+2.3})</td>
<td>(11.5_{-2.2}^{+2.3})</td>
</tr>
<tr>
<td>( F_{f_2p}(8.5);(\times 10^{-2}) )</td>
<td>(6.3_{-1.0}^{+1.1})</td>
<td>(6.3_{-1.0}^{+1.1})</td>
<td>(6.3_{-1.0}^{+1.1})</td>
</tr>
<tr>
<td>( F_{f_2p}(12.5);(\times 10^{-2}) )</td>
<td>(4.6_{-1.7}^{+1.9})</td>
<td>(4.6_{-1.7}^{+1.9})</td>
<td>(4.6_{-1.7}^{+1.9})</td>
</tr>
<tr>
<td>( \phi_{f_2D1} (^\circ) )</td>
<td>(33_{-17}^{+28})</td>
<td>(177_{-27}^{+27})</td>
<td>(112_{-35}^{+33})</td>
</tr>
<tr>
<td>( \phi_{f_2D2} (^\circ) )</td>
<td>(199_{-75}^{+34})</td>
<td>(218_{-29}^{+29})</td>
<td>(209_{-35}^{+30})</td>
</tr>
<tr>
<td>( \phi_{f_1D1} (^\circ) )</td>
<td>(137_{-34}^{+34})</td>
<td>(328_{-30}^{+30})</td>
<td>(183_{-30}^{+29})</td>
</tr>
<tr>
<td>( \phi_{f_1D2} (^\circ) )</td>
<td>(166_{-32}^{+30})</td>
<td>(180_{-29}^{+29})</td>
<td>(162_{-29}^{+29})</td>
</tr>
<tr>
<td>( f_S (\sqrt{\text{nb GeV}^2};(\times 10^{-2}) )</td>
<td>(1.3_{-0.1}^{+0.05})</td>
<td>(0.9_{-0.1}^{+0.05})</td>
<td>(0 (\text{fixed}))</td>
</tr>
<tr>
<td>( g_S (\text{GeV}) )</td>
<td>(0.10_{-0.24}^{+0.05})</td>
<td>(0.06_{-0.24}^{+0.05})</td>
<td>(0 (\text{fixed}))</td>
</tr>
<tr>
<td>( \phi_{BW} (^\circ) )</td>
<td>(297_{-21}^{+21})</td>
<td>(150_{-21}^{+21})</td>
<td>(0 (\text{fixed}))</td>
</tr>
<tr>
<td>( a_S (\sqrt{\text{nb}}; (\times 10^{-3}) )</td>
<td>(0 (\text{fixed}))</td>
<td>(4.3_{-5.9}^{+12.5})</td>
<td>(2.2_{-3.0}^{+5.7})</td>
</tr>
<tr>
<td>( b_S )</td>
<td>(0 (\text{fixed}))</td>
<td>(19.6_{-4.1}^{+4.6})</td>
<td>(21.9_{-4.0}^{+6.0})</td>
</tr>
<tr>
<td>( c_S )</td>
<td>(0 (\text{fixed}))</td>
<td>(0.00_{-0.06}^{+0.06})</td>
<td>(0.00_{-0.05}^{+0.05})</td>
</tr>
<tr>
<td>( \phi_{BS} (^\circ) )</td>
<td>(0 (\text{fixed}))</td>
<td>(99_{-21}^{+19})</td>
<td>(311_{-20}^{+18})</td>
</tr>
</tbody>
</table>
The six processes; in total ~20 peaks

Charged vs Neutral $\pi\pi$

Three neutral-pair processes
$\pi^0\pi^0$, $\eta\pi^0$, $\eta\eta$

Horizontal axis:

$W$ -- $\gamma\gamma$ c.m. energy = invariant mass of the two-meson system

$W<\sim2.5$ GeV: Dominated by resonances
$W>\sim2.5$ GeV: (Negative) Power law works + ($\chi_c$ charmonia)
The tensor-meson triplet, $f_2(1270)$, $a_2(1320)$, $f_2'(1525)$

$f_2(1270)$: The largest peak in $\pi^+\pi^-$ and $\pi^0\pi^0$. Also seen in $\eta$  
$a_2(1320)$: Large peak in $\eta\pi^0$  
$f_2'(1525)$: Large peak in $\eta\eta$, $K^+K^-$, and $K^0_SK^0_S$
W-dependences at high energies

\[ W \equiv W_{\gamma\gamma} \equiv \sqrt{S} \]  
Collision’s c.m. energy

Assume or expect \( \sigma(W) \sim W^{-n} \)

Fitted and reproduced
Slope parameter \( n \) different among the reactions

Charmonium contributions not included/removed

\( \pi^+\pi^- \)
\( K^+K^- \)
\( \eta\pi^0 \)
\( \pi^0\pi^0 \)
\( \eta\eta \)

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Cross sections and their ratios

| Process                  | $n$         | $W$(GeV) | $|\cos \theta^*|$ | BL | BC | DKV |
|--------------------------|-------------|----------|-------------------|----|----|-----|
| $K^0_S K^0_S$            | 11.0 ± 0.4 ± 0.4 | 2.4 - 4.0 | < 0.8            | 10 |    |     |
| $\pi^+\pi^-$            | 7.9 ± 0.4 ± 1.5 | 3.0 - 4.1 | < 0.6            | 6  | 6  |     |
| $K^+ K^-$                | 7.3 ± 0.3 ± 1.5 | 3.0 - 4.1 | < 0.6            | 6  | 6  |     |
| $\pi^0\pi^0$            | 8.0 ± 0.5 ± 0.4 | 3.1 - 4.1 | < 0.8            | 10 |    |     |
| $\eta\pi^0$             | 10.5 ± 1.2 ± 0.5 | 3.1 - 4.1 | < 0.8            | 10 |    |     |
| $\eta\eta$              | 7.8 ± 0.6 ± 0.4 | 2.4 - 3.3 | < 0.8            | 10 |    |     |
| $K^+ K^- / \pi^+\pi^-$  | 0.89 ± 0.04 ± 0.15 | 3.0 - 4.1 | < 0.6            | 2.3| 1.06|     |
| $K_S K_S / K^+ K^-$     | ~ 0.10 to ~ 0.03 | 2.4 - 4.0 | < 0.6            | 0.005 | 2/25 |
| $\pi^0\pi^0 / \pi^+\pi^-$ | 0.32 ± 0.03 ± 0.06 | 3.1 - 4.1 | < 0.6            | 0.04-0.07 | 0.5 |
| $\eta\pi^0 / \pi^0\pi^0$ | 0.48 ± 0.05 ± 0.04 | 3.1 - 4.0 | < 0.8            | 0.24$R_f$(0.46$R_f$)‡ |
| $\eta\eta / \pi^0\pi^0$ | 0.37 ± 0.02 ± 0.03 | 2.4 - 3.3 | < 0.8            | 0.36$R_f^2$(0.62$R_f^2$)‡ |

† Exclude $\chi_{cJ}$ region, 3.3 - 3.6 GeV.
‡ Assuming $\eta$ is a member of SU(3) octet (superposition of octet and singlet with mixing angle of $\theta_p = -18^\circ$).
$R_f$ is a ratio of decay constants, $f_\eta^2/f_{\pi^0}^2$.

- $n$ ranges 7 to 11. Close or not far from QCD prediction of 6 and 10.
- Cross section ratios tend to be constant above 3 GeV.
Angular dependence

\[ \gamma \gamma \rightarrow \pi^0 \pi^0 \]

\[ \frac{d\sigma}{d |\cos \theta^*|} \propto \sin^{-4} \theta^* \] is predicted by the q\bar{q}-meson model and perturbative QCD

- Fit to \( \sin^{-4} \theta^* + b \cos \theta^* \)
- \( b \) becomes constant above 3.2 GeV.

| mode   | \( \alpha \) in \( \sin^{-\alpha} \theta^* \) | GeV  | \( |\cos \theta^*| \)   |
|--------|---------------------------------------------|------|-------------------------|
| \( K_S K_S \) | 3 – 8                                       | 2.6 - 3.3 | < 0.8                   |
| \( \pi^+ \pi^- \) | Good agreement with 4                       | 3.0 - 4.1 | < 0.6                   |
| \( K^+ K^- \) | Good agreement with 4                       | 3.0 - 4.1 | < 0.6                   |
| \( \pi^0 \pi^0 \) | Better agreement with \( \sin^{-4} \theta^* + b \cos \theta^* \) | 2.4 - 4.1 | < 0.8                   |
|        | Approaches \( \sin^{-4} \theta^* \) above 3.1 GeV |      |                         |
| \( \eta \pi^0 \) | Good agreement with 4 above 2.7 GeV        | 3.1 - 4.1 | < 0.8                   |
| \( \eta \eta \) | Poor agreement with 4                      | 2.4 - 3.3 | < 0.9                   |
|        | Close to 6 above 3 GeV                     |      |                         |
|        | Exclude \( \uparrow \chi_{cJ} \) region, 3.3 - 3.6 GeV |      |                         |

Summarized by H.Nakazawa, Hadron2013

DIS2018, Apr.2018
Scalars in the 1.2 – 1.6 GeV region

- Hadron experiments report a wide $f_0(1370)$ and a narrow $f_0(1500)$.
- Some of previous two-photon measurements provide a hint of $f_0(1100-1400) \rightarrow \pi\pi$ under the huge peak of $f_2(1270)$
- Belle’s $\pi^0\pi^0$ measurement reports $f_0(1470)$. May be visible in the line shape.
  → favorable to the narrow $f_0(1500)$, but also consistent with $f_0(1370)$.

\[
\begin{array}{c|c|c}
\hline
\text{Parameter} & \text{Belle (\(\pi^0\pi^0\))} & \text{Crystal Ball} & \text{Unit} \\
\hline
\text{Mass} & 1470^{+6+72}_{-7-255} & 1250 & \text{MeV/c}^2 \\
\Gamma_{\text{tot}} & 90^{+2+50}_{-1-22} & 268 \pm 70 & \text{MeV} \\
\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0) & 11^{+4+603}_{-2-7} & 430 \pm 80 & \text{eV} \\
\hline
\end{array}
\]
$f_2(1270)$-$a_2(1320)$ interference in $K\bar{K}$

Constructive interference
$f_2(1270)+a_2(1320)$ in $K^+K^-$

Destructive interference
$f_2(1270)-a_2(1320)$ in $K^0\bar{K}^0$

Explained by a phase relation in isospin composition

**π⁰ Transition Form Factor (TFF)**

\[ \gamma\gamma^* \rightarrow \pi^0 \]

Coupling of neutral pion with two photons

Good test for QCD at high \( Q^2 \)

**Single-tag \( \pi^0 \) production in two-photon process**

- with a large-\( Q^2 \) and a small-\( Q^2 \) photon

Theoretically calculated from pion distribution amplitude and decay constant

\[
F(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int T_H(x, Q^2, \mu)\phi_\pi(x, \mu)dx
\]

BaBar has reported a significant deviation from the expectation.

**Measurement:**

\[
|F(Q^2)|^2 = |F(Q^2,0)|^2 = \frac{(d\sigma/dQ^2)}{(2A(Q^2))}
\]

\[
|F(0,0)|^2 = 64\pi\Gamma_{\gamma\gamma}/\{(4\pi\alpha)^2m_R^3\}
\]

Detects e (tag side) and \( \pi^0 \)

\( Q^2 = 2EE'(1 - \cos \theta) \) from energy and polar angle of the tagged electron

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**PRD 86, 092007 (2012)**

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Comparisons with Previous Measurements and Fits

Fit A (suggested by BaBar)

\[ Q^2|F(Q^2)| = A \left( \frac{Q^2}{10\text{GeV}^2} \right) ^\beta \]

BaBar:

\[ A = 0.182 \pm 0.002 \ (\pm 0.004) \text{ GeV} \]
\[ \beta = 0.25 \pm 0.02 \]

Belle:

\[ A = 0.169 \pm 0.006 \text{ GeV} \]
\[ \beta = 0.18 \pm 0.05 \]
\[ \chi^2/\text{ndf} = 6.90/13 \sim 1.5\sigma \text{ difference from BaBar} \]

Fit B (with an asymptotic parameter)

\[ Q^2|F(Q^2)| = BQ^2/(Q^2+C) \]

Belle:

\[ B = 0.209 \pm 0.016 \text{ GeV} \]
\[ C = 2.2 \pm 0.8 \text{ GeV}^2 \]
\[ \chi^2/\text{ndf} = 7.07/13 \]

B is consistent with the QCD value (0.185GeV)

No rapid growth above \( Q^2 > 9\text{GeV}^2 \) is seen in Belle result.

\( \sim 2.3\sigma \) difference between Belle and BaBar in 9 – 20 GeV²

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BaBar, PRD 80, 052002 (2009)