D-Mixing and search for CPV at Belle

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- Introduction
- WS decays
- Decays to CP eigenstates
- Self-conjugate decays
- Conclusions
Mixing

- Flavor eigenstates $\neq$ mass eigenstates (with $m_{1,2}$, $\Gamma_{1,2}$)

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle, \quad p^2 + q^2 = 1$$

- $D^0$ at $t = 0$ evolves as:

$$|D^0(t)\rangle = e^{-(\Gamma/2+im)t}[\cosh(\frac{y + ix}{2}\Gamma t)|D^0\rangle + \frac{q}{p}\sinh(\frac{y + ix}{2}\Gamma t)|\bar{D}^0\rangle]$$

\[
x = \frac{\Delta m}{\Gamma} \quad y = \frac{\Delta \Gamma}{2 \Gamma}
\]

- $|x|, |y| \ll 1$:

$$\frac{dN_{D^0 \rightarrow f}}{dt} \propto |\langle f | \mathcal{H} | D^0(t) \rangle|^2 = e^{-\Gamma t} |\langle f | \mathcal{H} | D^0 \rangle + \frac{q}{p} (\frac{y + ix}{2}\Gamma t)|f | \mathcal{H} | \bar{D}^0 \rangle|^2$$

- Decay time distribution of different final states sensitive to different combinations of mixing parameters $x$ and $y$. 
CP violation

\[ |D_{1,2}⟩ = p|D^0⟩ ± q|\bar{D}^0⟩ \]

- \( q/p \neq 1 \Rightarrow \text{indirect CP violation} \)
- \( |q/p| = |q|/|p| \cdot e^{i\phi} \):
  - \( |q/p| \neq 1 \Rightarrow \text{CP violation in mixing} \)
  - \( \phi \neq 0(\pi) \Rightarrow \text{CP violation in interference of decays w/ and w/o mixing} \)
- \( |\mathcal{A}(D^0 \rightarrow f)|^2 \neq |\mathcal{A}(\bar{D}^0 \rightarrow \bar{f})|^2 \Rightarrow \text{direct CP violation} \)
Experimental method

- $D^{*+} \rightarrow \pi^+ D^0$
  - flavor tagging by $\pi_{slow}$ charge
  - background suppression

- $D^0$ proper decay time $t$ measurement:
  $$t = \frac{l_{dec}}{c\beta\gamma}, \quad \beta\gamma = \frac{p_{D^0}}{M_{D^0}}$$
  $\sigma_t$ ... decay-time uncertainty
  (from vtx cov. matrices)

- Measurements performed at $\Upsilon(4S)$
  - to reject $D^{*+}$ from $B$ decays:
    $$p_{D^{*+}}^{CMS} > 2.5 \text{ GeV/c}$$

- Observables:
  $$m = m(K\pi)$$
  $$q = m(K\pi\pi_s) - m(K\pi) - m_\pi$$
Wrong Sign decays $K^{+}\pi^{-}$

PRL 96, 151801 (2006)

- Wrong sign (WS) final state:
  - via doubly Cabibbo suppressed decay (DCS) or via mixing

Proper decay time distribution of WS events (assuming negligible CPV)

$$\frac{dN}{dt} \propto [R_D + y' \sqrt{R_D} (\Gamma t) + \frac{x'^2 + y'^2}{4} (\Gamma t)^2] e^{-\Gamma t}$$

- DCS
- interference
- mixing

$R_D$ ratio of DCS/CF decay rates

$x' = x \cos \delta + y \sin \delta$

$y' = y \cos \delta - x \sin \delta$

$\delta$ strong phase between DCS and CF
$D^0 \rightarrow K^+\pi^- (400 \text{ fb}^{-1})$

- **Search for CPV**
  - Fit $D^0$ and $\bar{D}^0$ samples separately $\Rightarrow R_D^\pm$, $x'^\pm$, $y'^\pm$

- **CPV in DCS decays:**
  \[
  A_D = \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-}
  \]

- **CPV in mixing and interference** $\rightarrow$ by solving 4 equations for 4 unknowns:
  \[
  x'^\pm = \left(1 \pm \frac{1}{2}A_M\right) \cdot (x' \cos \phi \pm y' \sin \phi)
  \]
  \[
  y'^\pm = \left(1 \pm \frac{1}{2}A_M\right) \cdot (y' \cos \phi \mp x' \sin \phi)
  \]

  $\rightarrow x'$, $y'$, $\phi$, $|q/p| = 1 + \frac{1}{2}A_M$
Results

♦ DCS/CF ratio:
\[ R_D = (0.364 \pm 0.017)\% \]

♦ Mixing:
\[ x' = (0.18^{+0.21}_{-0.23}) \times 10^{-3} \]
\[ y' = (0.6^{+4.0}_{-3.9}) \times 10^{-3} \]

\[ \rightarrow \text{no mixing point at } 2\sigma \]

♦ Search for CPV:
\[ A_D = (2.3 \pm 4.7)\% \]
\[ A_M = 0.67 \pm 1.2 \]
\[ |\phi| = 0.16 \pm 0.44 \]

\[ \rightarrow \text{consistent with no CPV} \]
$D^0 \rightarrow K^+ K^-, \pi^+ \pi^- (540 \text{ fb}^{-1})$

Decays to CP-even eigenstates $K^+ K^-, \pi^+ \pi^-$

PRL 98, 211803 (2007)

✦ Measurement of lifetime difference between $D^0 \rightarrow K^- \pi^+$ and $K^+ K^-, \pi^+ \pi^-$

▷ mixing parameter: $y_{CP} = \frac{\tau(K^- \pi^+)}{\tau(K^+ K^-)} - 1$

▷ in CP conservation limit: $y_{CP} = y = \Delta \Gamma / 2 \Gamma$

✦ If CP not conserved, difference in lifetimes of $D^0/\bar{D}^0 \rightarrow K^+ K^-, \pi^+ \pi^-$

▷ CP violating parameter: $A_{\Gamma} = \frac{\tau(\bar{D}^0 \rightarrow K^- K^+)-\tau(D^0 \rightarrow K^+ K^-)}{\tau(\bar{D}^0 \rightarrow K^- K^+)+\tau(D^0 \rightarrow K^+ K^-)}$

▷ $y_{CP} = y \cos \phi - \frac{1}{2} A_M x \sin \phi$

▷ $A_{\Gamma} = \frac{1}{2} A_M y \cos \phi - x \sin \phi$ (S. Bergmann et.al., PLB 486, 418 (2000))

PLB 670, 190 (2008)

✦ Measurement of CP-violating asymmetry $A_{CP}$

$A_{CP}^f = \frac{B(D^0 \rightarrow f) - B(\bar{D}^0 \rightarrow \bar{f})}{B(D^0 \rightarrow f) + B(\bar{D}^0 \rightarrow \bar{f})}$

$A_{CP}^f = a_d^f + a_{ind} = a_d^f - A_{\Gamma}$
$D^0 \rightarrow K^+ K^-, \pi^+ \pi^- \ (540 \text{ fb}^{-1})$

- **Data samples: signal yields (purities)**

<table>
<thead>
<tr>
<th>channel</th>
<th>$KK$</th>
<th>$K\pi$</th>
<th>$\pi\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal</td>
<td>110k</td>
<td>1.2M</td>
<td>50k</td>
</tr>
<tr>
<td>purity</td>
<td>98%</td>
<td>99%</td>
<td>92%</td>
</tr>
</tbody>
</table>

- **Background estimated from sidebands in $m$**
- **Resolution function: decay mode and run period dependent**
- **Simultaneous $KK/\pi\pi/K\pi$ binned maximum likelihood fit**

quality of fit: $\chi^2 = 1.084 \ (289)$
$D^0 \rightarrow K^+K^-$, $\pi^+\pi^-$ ($540 \text{ fb}^{-1}$)

Results

<table>
<thead>
<tr>
<th></th>
<th>$y_{CP}$ (%)</th>
<th>$A_\Gamma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$KK$</td>
<td>1.25$\pm$0.39$\pm$0.28</td>
<td>0.15$\pm$0.34$\pm$0.16</td>
</tr>
<tr>
<td>$\pi\pi$</td>
<td>1.44$\pm$0.57$\pm$0.42</td>
<td>$-0.28 \pm 0.52 \pm 0.30$</td>
</tr>
<tr>
<td>$KK + \pi\pi$</td>
<td>1.31$\pm$0.32$\pm$0.25</td>
<td>0.01$\pm$0.30$\pm$0.15</td>
</tr>
</tbody>
</table>

Evidence for $D^0 - \overline{D}^0$ mixing (regardless of possible CPV)

$y_{CP} = (1.31 \pm 0.32 \pm 0.25)$ %

> $3\sigma$ above zero ($4.1\sigma$ stat. only)

$A_\Gamma = (0.01 \pm 0.30 \pm 0.15)$ %

no evidence for CP violation
Search for CP-violating asymmetry $A_{CP}$

- Measured asymmetry

\[ A^{reco} = A^{D_{FB}}_{FB} + A^{f}_{CP} + A^{\pi}_{\epsilon} \]

- Asymmetry of slow pion efficiency ($A^{\pi}_{\epsilon}$) can be measured using tagged and untagged $D^0 \rightarrow K^-\pi^+$

\[ A^{\text{tagrec}}_{rec} = A^{FB}_{FB} + A^{K\pi}_{CP} + A^{K\pi}_{\epsilon} + A^{\pi}_{\epsilon} \]

\[ A^{\text{untagrec}}_{rec} = A^{FB}_{FB} + A^{K\pi}_{CP} + A^{K\pi}_{\epsilon} \]

- Efficiency corrected asymmetry:

\[ A^{reco}_{corr} = A^{reco}_{rec} - A^{\pi}_{\epsilon} = A^{FB}_{FB} + A^{f}_{CP} \]

- Forward-backward asymmetry is an odd function of \( \cos \theta^* \)

\[ A_{CP} = \frac{A^{reco}_{corr}(\cos \theta^*) + A^{reco}_{corr}(-\cos \theta^*)}{2} \]

\[ A_{FB} = \frac{A^{reco}_{corr}(\cos \theta^*) - A^{reco}_{corr}(-\cos \theta^*)}{2} \]
Results

\[ D^0 \rightarrow K^+ K^-, \pi^+ \pi^- (540 \text{ fb}^{-1}) \]

\[ A_{\text{CP}}^{KK} = (-0.43 \pm 0.30 \pm 0.11)\% \]
\[ A_{\text{CP}}^{\pi\pi} = (+0.43 \pm 0.52 \pm 0.12)\% \]
\[ \rightarrow \text{consistent with no CPV} \]

Direct CPV: \[ a_d^f = A_{\text{CP}}^f + A_\Gamma \]

\[ a_d^{KK} = (-0.42 \pm 0.42 \pm 0.19)\% \]
\[ a_d^{\pi\pi} = (+0.44 \pm 0.60 \pm 0.19)\% \]
\[ \rightarrow \text{no sign of direct CPV} \]
$D^0 \rightarrow \phi K^0_s \ (673 \text{ fb}^{-1})$

Decays to CP-odd eigenstate $\phi K^0_s$

to be submitted to PRD

✦ Measurement of lifetime difference between CP-even and CP-odd eigenstates

✦ $m(K^+K^-)$ dependent CP mixture:
  ▶ peak region: mainly CP-odd ($\phi(1020)$)
  ▶ sideband: mainly CP-even ($a_0(980)$)

✦ Decay rate (no CPV):

$$\frac{dN}{dt} \propto a_1(s_0)e^{-\frac{t}{\tau}(1+y)} + a_2(s_0)e^{-\frac{t}{\tau}(1-y)}$$

✦ By measuring effective lifetimes in the peak region (ON) and in sideband (OFF)

$$y_{CP} = \frac{1}{f_{ON} - f_{OFF}} \cdot \frac{\tau_{ON} - \tau_{OFF}}{\tau_{ON} + \tau_{OFF}}$$

$f_{ON}, f_{OFF}$ CP-even fractions, obtained from Dalitz model

✦ Topologically equal events in ON and OFF regions $\rightarrow$ reduced effects of resolution function.
$D^0 \rightarrow \phi K^0_s$ ($673 \text{ fb}^{-1}$)

- Un-tagged data sample used to increase statistics

<table>
<thead>
<tr>
<th>region</th>
<th>ON</th>
<th>OFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal</td>
<td>72k</td>
<td>62k</td>
</tr>
<tr>
<td>purity</td>
<td>97%</td>
<td>91%</td>
</tr>
</tbody>
</table>

- Background estimated from sidebands in $(m_{D^0}, m_{K^0_s})$ plane
- $f_{ON}, f_{OFF}$ from fit to $m(K^+K^-)$ using 8-resonance Dalitz model
- $\tau_{ON}, \tau_{OFF}$ determined from mean proper decay times of all events and background events:

$$\tau_R = \frac{<t>_R - (1 - p^R) <t>^R_b}{p^R}, \quad R = \{ON, OFF\}$$

Results

$$y_{CP} = (0.11 \pm 0.61 \pm 0.52) \%$$
$D^0 \rightarrow K_s^0 \pi^+\pi^-$ \textit{Dalitz} (540 fb$^{-1}$)

Self-conjugate decays $K_s^0 \pi^+\pi^-$

PRL 99, 131803 (2007)

 다양한 decays identified through Dalitz plot analysis

\begin{itemize}
  \item CF: $D^0 \rightarrow K^{*-}\pi^+$
  \item DCS: $D^0 \rightarrow K^{*+}\pi^-$
  \item CP: $D^0 \rightarrow \rho^0 K_s^0$
\end{itemize}

→ relative phases can be determined (unlike $D^0 \rightarrow K^+\pi^-$)

\begin{itemize}
  \item Matrix element is Dalitz space dependent:
\end{itemize}

$$\mathcal{M}(m_2^-, m_2^+, t) = \mathcal{A}(m_2^-, m_2^+) \frac{e_1(t) + e_2(t)}{2} + \frac{q}{p} \mathcal{A}(m_2^-, m_2^+) \frac{e_1(t) - e_2(t)}{2}$$

where $m_2^\pm = m^2(K_s^0\pi^\pm)$ and $e_{1,2}(t) = e^{-i(m_{1,2}-i\Gamma_{1,2}/2)t}$

\begin{itemize}
  \item Amplitudes $\mathcal{A}(\mathcal{A})$ for $D^0(\overline{D}^0)$ decays parametrized as a sum of quasi-two-body amplitudes + non-resonant contribution
  \item Decay rate $dN/dt \propto |\mathcal{M}(m_2^-, m_2^+, t)|^2$ contains terms
    \begin{align*}
      &\exp(-\Gamma t) \cos(x\Gamma t), \quad \exp(-\Gamma t) \sin(x\Gamma t), \quad \exp[-(1 \pm y)\Gamma t]
    \end{align*}
  \item With time-dependent Dalitz plot analysis both mixing parameters ($x$ and $y$) can be measured.
\end{itemize}
$D^0 \rightarrow K^*_s \pi^+ \pi^-$ Dalitz (540 fb$^{-1}$)

- Signal yield and purity

<table>
<thead>
<tr>
<th>signal</th>
<th>purity</th>
</tr>
</thead>
<tbody>
<tr>
<td>534000</td>
<td>95%</td>
</tr>
</tbody>
</table>

- Dalitz projection of a 3D fit (unbinned max. likelihood)

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
<th>Fit fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(892)^-$</td>
<td>1.629 ± 0.005</td>
<td>134.3 ± 0.3</td>
<td>0.6227</td>
</tr>
<tr>
<td>$K^0(1430)^-$</td>
<td>2.12 ± 0.02</td>
<td>−0.9 ± 0.5</td>
<td>0.0724</td>
</tr>
<tr>
<td>$K^*_2(1430)^-$</td>
<td>0.87 ± 0.01</td>
<td>−47.3 ± 0.7</td>
<td>0.0133</td>
</tr>
<tr>
<td>$K^*(1410)^-$</td>
<td>0.65 ± 0.02</td>
<td>111 ± 2</td>
<td>0.0048</td>
</tr>
<tr>
<td>$K^*(1680)^-$</td>
<td>0.60 ± 0.05</td>
<td>147 ± 5</td>
<td>0.0002</td>
</tr>
<tr>
<td>$K^*(892)^+$</td>
<td>0.152 ± 0.003</td>
<td>−37.5 ± 1.1</td>
<td>0.0054</td>
</tr>
<tr>
<td>$K^0(1430)^+$</td>
<td>0.541 ± 0.013</td>
<td>91.8 ± 1.5</td>
<td>0.0047</td>
</tr>
<tr>
<td>$K^*_2(1430)^+$</td>
<td>0.276 ± 0.010</td>
<td>−106 ± 3</td>
<td>0.0013</td>
</tr>
<tr>
<td>$K^*(1410)^+$</td>
<td>0.333 ± 0.016</td>
<td>−102 ± 2</td>
<td>0.0013</td>
</tr>
<tr>
<td>$K^*(1680)^+$</td>
<td>0.73 ± 0.10</td>
<td>103 ± 6</td>
<td>0.0004</td>
</tr>
<tr>
<td>ρ(770)</td>
<td>1 (fixed)</td>
<td>0 (fixed)</td>
<td>0.2111</td>
</tr>
<tr>
<td>ω(782)</td>
<td>0.0380 ± 0.0006</td>
<td>115.1 ± 0.9</td>
<td>0.0063</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>0.380 ± 0.002</td>
<td>−147.1 ± 0.9</td>
<td>0.0452</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>1.46 ± 0.04</td>
<td>98.6 ± 1.4</td>
<td>0.0162</td>
</tr>
<tr>
<td>$f_2(1270)$</td>
<td>1.43 ± 0.02</td>
<td>−13.6 ± 1.1</td>
<td>0.0180</td>
</tr>
<tr>
<td>$\rho(1450)$</td>
<td>0.72 ± 0.02</td>
<td>40.9 ± 1.9</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>1.387 ± 0.018</td>
<td>−147 ± 1</td>
<td>0.0914</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.267 ± 0.009</td>
<td>−157 ± 3</td>
<td>0.0088</td>
</tr>
<tr>
<td>NR</td>
<td>2.36 ± 0.05</td>
<td>155 ± 2</td>
<td>0.0615</td>
</tr>
</tbody>
</table>
$D^0 \rightarrow K^0_s \pi^+\pi^-$ Dalitz (540 fb$^{-1}$)

Results

Assuming CP conservation

$$x = 0.80 \pm 0.29_{-0.16}^{+0.13} \%$$
$$y = 0.33 \pm 0.24_{-0.10}^{+0.10} \%$$

most stringent limits on $x$ up to now
Cleo, PRD 72, 012001 (2005):
$$x = 1.8 \pm 3.4 \pm 0.6\%$$
$$y = -1.4 \pm 2.5 \pm 0.9\%$$

Search for CP violation

✦ Dalitz plot fit separately for $D^0$ and $\bar{D}^0$
✦ fit parameters consistent for both samples
  → no direct CPV
✦ parameters $|q/p|$ and $\phi = \text{arg}(q/p)$
  consistent with CP conservation

$$|q/p| = 0.86_{-0.29}^{+0.30+0.10}$$
$$\phi = (-0.24_{-0.30}^{+0.28} \pm 0.09)$$

Time projection of fit

$\tau = 409.9 \pm 0.9$ fs
→ consistent with PDG

95% C.L. contours
Conclusions

✦ Measurements of D-mixing at Belle from recent years as well as searches for CPV have been presented.
✦ In 2007 the first evidence for D-mixing found in decays to CP eigenstates.
✦ From time-dependent Dalitz plot analysis the most sensitive measurement of $\chi$ up to now.
✦ CPV: no evidence found so far.
✦ Till the end of Belle data taking (next spring) expect to reach $1\text{ ab}^{-1}$
  ▶ these measurements will be updated
  ▶ planned to analyse also other perspective decay modes
    $(\pi^+\pi^-\pi^0, K^0_s K^+K^-, K^+\pi^-\pi^0, ...)$
Statistical method

✦ $y_{CP}$ and $A_\Gamma$ can be determined from mean of the timing distributions (e.g. without fitting the data), and the error from r.m.s

✦ Assumptions:
  ▶ timing distribution is a convolution of exponential with some resolution function + some background
  ▶ resolution function offsets of final states are the same and small

$$P(t) = p \frac{1}{\tau} e^{-t/\tau} \ast R_s(t) + (1-p)B(t) \Rightarrow \langle t \rangle = p(\tau + t_0) + (1-p)\langle t \rangle_b$$

$$\tau + t_0 = \frac{\langle t \rangle - (1-p)\langle t \rangle_b}{p} = \langle t \rangle_s$$

✦ In lifetime difference $t_0$ cancels, thus if $t_0 \ll \tau$

$$y_{CP} = \frac{\langle t \rangle_{K\pi} - \langle t \rangle_{KK}}{\langle t \rangle_{KK}}$$

✦ Result with this method for $D^0 \rightarrow K^+K^-, \pi^+\pi^-$:

$$y_{CP} = (1.35 \pm 0.33_{stat})\%$$
Systematics of $D^0 \to K^+ K^-, \pi^+ \pi^-$

<table>
<thead>
<tr>
<th>source</th>
<th>$y_{CP}$</th>
<th>$A_{\Gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance</td>
<td>0.12%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Equal $t_0$</td>
<td>0.14%</td>
<td>0.08%</td>
</tr>
<tr>
<td>Mass window position</td>
<td>0.04%</td>
<td>0.003%</td>
</tr>
<tr>
<td>Signal/sideband background differences</td>
<td>0.09%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Opening angle distributions</td>
<td>0.02%</td>
<td></td>
</tr>
<tr>
<td>Background distribution $B(t)$</td>
<td>0.07%</td>
<td>0.07%</td>
</tr>
<tr>
<td>(A)symmetric resolution function</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Selection variation</td>
<td>0.11%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Binning of $t$ distribution</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Sum in quadrature</td>
<td>0.25%</td>
<td>0.15%</td>
</tr>
</tbody>
</table>

Systematic uncertainties in $A_{CP}$

<table>
<thead>
<tr>
<th>Source</th>
<th>$D^0 \to K^+ K^-$</th>
<th>$D^0 \to \pi^+ \pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal counting</td>
<td>0.04%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Slow pion corrections</td>
<td>0.10%</td>
<td>0.10%</td>
</tr>
<tr>
<td>$A_{CP}$ extraction</td>
<td>0.03%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Sum in quadrature</td>
<td>0.11%</td>
<td>0.12%</td>
</tr>
</tbody>
</table>
Systematics of $D^0 \rightarrow \phi K_s^0$

<table>
<thead>
<tr>
<th>Source</th>
<th>Systematic error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution function offset difference $t_{0}^{OFF} - t_{0}^{ON}$</td>
<td>±0.38</td>
</tr>
<tr>
<td>Estimation of $\langle t \rangle_b$</td>
<td>±0.10</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ background</td>
<td>±0.07</td>
</tr>
<tr>
<td>Selection of sideband</td>
<td>±0.05</td>
</tr>
<tr>
<td>Variation of selection criteria</td>
<td>±0.30</td>
</tr>
<tr>
<td>Fitting procedure</td>
<td>±0.10</td>
</tr>
<tr>
<td>Proper decay time range and binning</td>
<td>±0.07</td>
</tr>
<tr>
<td>Dalitz model</td>
<td>±0.01</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>±0.52</td>
</tr>
</tbody>
</table>
### Systematics of $D^0 \rightarrow K^0_s \pi^+\pi^-$ Dalitz

#### Experimental

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta x$ (%)</th>
<th>$\Delta y$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event selection</td>
<td>+0.076</td>
<td>+0.018</td>
</tr>
<tr>
<td></td>
<td>-0.001</td>
<td>-0.078</td>
</tr>
<tr>
<td>Dalitz dep. effi.</td>
<td>+0.004</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>+0.041</td>
<td>+0.077</td>
</tr>
<tr>
<td>Background</td>
<td>-0.068</td>
<td>-0.086</td>
</tr>
<tr>
<td>Total</td>
<td>+0.09</td>
<td>+0.08</td>
</tr>
<tr>
<td></td>
<td>-0.07</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

### Model dependence

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta x$ (%)</th>
<th>$\Delta y$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M&amp;\Gamma$ errors</td>
<td>±0.020</td>
<td>±0.010</td>
</tr>
<tr>
<td>$F_r = F_D = 1$</td>
<td>-0.031</td>
<td>+0.006</td>
</tr>
<tr>
<td>$\Gamma(q^2) = \text{const.}$</td>
<td>-0.051</td>
<td>-0.041</td>
</tr>
<tr>
<td>K-Matrix</td>
<td>±0.073</td>
<td>±0.058</td>
</tr>
<tr>
<td>No NR</td>
<td>-0.015</td>
<td>+0.003</td>
</tr>
<tr>
<td>No $K^*(1680)^+$</td>
<td>-0.003</td>
<td>-0.008</td>
</tr>
<tr>
<td>No $\rho(1450)$</td>
<td>-0.005</td>
<td>-0.006</td>
</tr>
<tr>
<td>$K^*_0(1430)$ DCS/CF</td>
<td>-0.103</td>
<td>+0.001</td>
</tr>
<tr>
<td>$K^*_2(1430)$ DCS/CF</td>
<td>+0.069</td>
<td>-0.025</td>
</tr>
<tr>
<td>$K^*(1410)$ DCS/CF</td>
<td>-0.016</td>
<td>+0.009</td>
</tr>
<tr>
<td>Total</td>
<td>+0.10</td>
<td>+0.06</td>
</tr>
<tr>
<td></td>
<td>-0.14</td>
<td>-0.08</td>
</tr>
</tbody>
</table>