$D^0 - \overline{D^0}$ Mixing and CP Violation in $D^0 \rightarrow K_{shh}$ measurements

Longke LI

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On behalf of the Belle Collaboration

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Outline

1 Introduction
   - $D^0 - \bar{D}^0$ mixing formalism
   - Experimental methods
   - $D^0$ self-conjugated decays

2 $D^0 \to K_{shh}$ measurements
   - Available charm samples
   - All published results’ table
   - Mixing and CPV at BaBar and Belle

3 New result in $K_S\pi^+\pi^-$ at Belle
   - Signal and backgrounds
   - Dalitz model and plot
   - Systematics estimation
   - Final result

4 Summary

- $A_{CP}$ measurement at CDF
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4 Summary
Mixing parameters

- Initial state \( (t=0) \): \( |\psi(t=0)\rangle = a(0)|D^0\rangle + b(0)|\bar{D}^0\rangle \)
  Evolve in time: \( |\psi(t)\rangle = a(t)|D^0\rangle + b(t)|\bar{D}^0\rangle \)

- Time evolution of \( D^0 -\bar{D}^0 \) system given by Schrödinger Equation.
  \[
i \frac{\partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \left( M - \frac{i}{2} \Gamma \right) \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}.
  \]
Mixing parameters

- Initial state (t=0): $|\psi(t = 0)\rangle = a(0)|D^0\rangle + b(0)|\bar{D}^0\rangle$
- Evolve in time: $|\psi(t)\rangle = a(t)|D^0\rangle + b(t)|\bar{D}^0\rangle$
- Time evolution of $D^0$-$\bar{D}^0$ system given by Schrödinger Equation.
  $$i \frac{\partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \left( M - i \frac{\Gamma}{2} \right) \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}.$$  
- Flavor eigenstate $\neq$ mass eigenstate: $D_{1,2}$ with $m_{1,2}$ and $\Gamma_{1,2}$.
  $$|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\bar{D}^0\rangle$$ with $p^2 + q^2 = 1$ (CPT conservation).
Mixing parameters

- Initial state ($t=0$): $|\psi(t = 0)\rangle = a(0)|D^0\rangle + b(0)|\bar{D}^0\rangle$
  Evolve in time: $|\psi(t)\rangle = a(t)|D^0\rangle + b(t)|\bar{D}^0\rangle$
- Time evolution of $D^0-\bar{D}^0$ system given by Schrödinger Equation:
  \[ i \frac{\partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \left( M - i \frac{\Gamma}{2} \right) \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}. \]
- Flavor eigenstate $\neq$ mass eigenstate: $D_{1,2}$ with $m_{1,2}$ and $\Gamma_{1,2}$.
  $|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\bar{D}^0\rangle$ with $p^2 + q^2 = 1$ (CPT conservation).
- **Mixing parameters:**
  \[ x \equiv \frac{m_1 - m_2}{\Gamma}, \quad y \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}. \]
- SM: Mixing in charm system strongly suppressed: $|x|, |y| \sim 1\%$
Time evolution and CP-violation

- Corresponding eigenvalues:
  \[ \lambda_{1,2} = m_{1,2} - \frac{i}{2} \Gamma_{1,2} = (M - \frac{i}{2} \Gamma) \pm \frac{q}{p} (M_{1,2} - \frac{i}{2} \Gamma_{1,2}) \]

- Proper time evolution:
  \[ |D_{1,2}(t)\rangle = e^{-i\lambda_{1,2} t}|D_{1,2}(t = 0)\rangle \]
Time evolution and CP-violation

- Corresponding eigenvalues:
  \[ \lambda_{1,2} = m_{1,2} - \frac{i}{2} \Gamma_{1,2} = (M - \frac{i}{2} \Gamma) \pm \frac{q}{p} (M_{1,2} - \frac{i}{2} \Gamma_{1,2}) \]

- \(|D_{1,2}\rangle\) proper time evolution: 
  \[ |D_{1,2}(t)\rangle = e^{-i\lambda_{1,2}t} |D_{1,2}(t = 0)\rangle \]

- Time evolution of initial pure flavor states\((t = 0)\):
  \[ |D^0(t)\rangle = \left[ |D^0\rangle \cosh\left(\frac{ix+y}{2} \Gamma t\right) + \frac{q}{p} |D^0\rangle \sinh\left(\frac{ix+y}{2} \Gamma t\right) \right] \times e^{-\frac{1}{2} (1 + \frac{im}{\Gamma})t} \]
Time evolution and CP-violation

- Corresponding eigenvalues:
  \[
  \lambda_{1,2} = m_{1,2} - \frac{i}{2} \Gamma_{1,2} = (M - \frac{i}{2} \Gamma) \pm \frac{q}{p} \left( M_{1,2} - \frac{i}{2} \Gamma_{1,2} \right)
  \]

- \(|D_{1,2}\rangle\) proper time evolution: 
  \[
  |D_{1,2}(t)\rangle = e^{-i\lambda_{1,2}t} |D_{1,2}(t = 0)\rangle
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- Time evolution of initial pure flavor states \((t = 0)\):
  \[
  |D^0(t)\rangle = \left[ |D^0\rangle \cosh\left( \frac{ix+y}{2} \Gamma t \right) + \frac{q}{p} |\bar{D}^0\rangle \sinh\left( \frac{ix+y}{2} \Gamma t \right) \right] \times e^{-\frac{1}{2}(1+im)\Gamma t}
  \]

Classification of CP-violation:
\[
A_{CP} = \frac{\Gamma(D\to f) - \Gamma(\bar{D}\to \bar{f})}{\Gamma(D\to f) + \Gamma(\bar{D}\to \bar{f})} = a_d^f + a_m^f + a_i^f
\]

1) \(a_d^f\) in decay: \(|A_f| \neq |\bar{A}_f|\);
2) \(a_m^f\) in mixing: \(|q/p| \neq 1\);
3) \(a_i^f\) in interference \((f = \bar{f})\): \(\phi = \arg\left( \frac{q}{p} \frac{\bar{A}_f}{A_f} \right) \neq 0\)

no direct-CPV: \(\phi = \arg\left( \frac{q}{p} \right)\)
Common experiment features

- Observables:
  - Invariant mass of $D^0$ daughter particles: $M$.
  - $Q = M_{D^0\pi_s} - M_{D^0} - M_{\pi}$ at Belle
  - $\Delta M = M_{K_S\pi^+\pi^-} - M_{K_S\pi^+\pi^-}$ at BaBar and CDF
- Use $D^0$ from $D^{*+} \rightarrow D^0\pi_s^+$
  - Charge of slow $\pi$: tagging the flavor of $D^0$
  - Background reduction
Observables:

- Invariant mass of $D^0$ daughter particles: $M$.
- $Q = M_{D^0\pi^s} - M_{D^0} - M_\pi$ at Belle
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Use $D^0$ from $D^{*+} \rightarrow D^0\pi^+$

- Charge of slow $\pi$: tagging the flavor of $D^0$
- Background reduction

At B-factories:

- $D^0$ proper decay time: $t = \frac{\ell_{\text{dec}}}{c\beta\gamma}$,
  $\beta\gamma = \frac{p_{D^0}}{M_{D^0}}$

- To eliminate $D^{*+}$ from $B$ decays:
  $p_{D^{*+}}^{CM} > 2.5(3.1)\text{GeV}/c$ for $\Upsilon(4S)(\Upsilon(5S))$ data.
## Decay modes and sensitivity

- **Methods/Precision/Measured parameters** depend on **Decay Modes**:

<table>
<thead>
<tr>
<th>Decay Modes</th>
<th>example</th>
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<tbody>
<tr>
<td>Semileptonic</td>
<td>$D^0 \rightarrow K^+ l \nu$</td>
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<td>Multi-body self-conjugated states</td>
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<td>A combination of those examples</td>
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Multi-body self-conjugated decays:

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>$BR \cdot \varepsilon$</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow K_S \pi^+ \pi^-$</td>
<td>100</td>
<td>22</td>
<td>20</td>
<td>★★★</td>
</tr>
<tr>
<td>$D^0 \rightarrow K_S K^+ K^-$</td>
<td>20</td>
<td>73</td>
<td>51</td>
<td>★★</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi^0 \pi^+ \pi^-$</td>
<td>15</td>
<td>100(default)</td>
<td>49</td>
<td>★</td>
</tr>
</tbody>
</table>

Taking into account of branch ratio and efficiency (@B-factory)
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   • $D^0$ – $\bar{D}^0$ mixing formalism
   • Experimental methods
   • $D^0$ self-conjugated decays

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   • Signal and backgrounds
   • Dalitz model and plot
   • Systematics estimation
   • Final result

4 Summary
Ref.: A. Zupanc@FPCP2011

**B-factories:**
- continuum production @ Υ(4S):
  \[ \sigma(c\bar{c}) \approx 1.3 \text{ nb} \]
- Belle: \( \sim 1.3 \cdot 10^9 \) \( c\bar{c} \) pairs
- Babar: \( \sim 0.7 \cdot 10^9 \) \( c\bar{c} \) pairs

**Tevatron:**
- \( pp \) @ \( \sim 2 \) TeV
- CDF: \( \sim 70 \cdot 10^9 \) \( D^0 \)’s

**Charm factories:**
- \( \psi(3770) \rightarrow D^0\overline{D^0}, \ D^+D^- \)
- CLEO: \( \sim 2.8 \cdot 10^6 \) \( D^0\overline{D^0} \) pairs
- BESIII: \( \sim 12 \cdot 10^6 \) \( D^0\overline{D^0} \) pairs

**LHC:**
- \( pp \) @ 7 TeV
- LHCb: first results
Self-conjugated decay $D^0 \rightarrow K_S h h$ (where $h = K$ or $\pi$) published measurements from all experiments:

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Year</th>
<th>Data</th>
<th>Decay Channel</th>
<th>Results</th>
<th>Ref.</th>
</tr>
</thead>
</table>
| CLEO  | 2005 | 9.0 fb$^{-1}$ | $D^0 \rightarrow K_S \pi^+ \pi^-$ | $x = (1.8^{+3.4}_{-3.2} \pm 0.4 \pm 0.4)$% 
$y = (-1.4^{+2.5}_{-2.4} \pm 0.8 \pm 0.4)$% | PRD72.012001 |
| Belle | 2007 | 540 fb$^{-1}$ | $D^0 \rightarrow K_S \pi^+ \pi^-$ | $x = (0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14})$% 
$y = (0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08})$% 
$|q/p| = 0.95^{+0.22+0.10}_{-0.20-0.09}$ 
$\text{arg}(q/p) = -0.035^{+0.17}_{-0.19} \pm 0.09$% | PRL99.121803 |
|      | 2009 | 673 fb$^{-1}$ | $D^0 \rightarrow K_S K^+ K^-$ | $y_{CP} = (+0.11 \pm 0.61 \pm 0.52)$% | PRD80.052006 |
| BaBar | 2010 | 469 fb$^{-1}$ | $D^0 \rightarrow K_S h^+ h^-$ | $x = (0.16 \pm 0.23 \pm 0.12 \pm 0.08)$% 
$y = (0.57 \pm 0.20 \pm 0.13 \pm 0.07)$% | PRL105.081803 |
|      |      |          | only $D^0 \rightarrow K_S \pi^+ \pi^-$ | $x = (+0.26 \pm 0.24)$% 
$y = (0.60 \pm 0.21)$% |               |
|      |      |          | only $D^0 \rightarrow K_S K^+ K^-$ | $x = (-1.36 \pm 0.21)$% 
$y = (0.44 \pm 0.57)$% |               |
| CDF   | 2012 | 6.0 fb$^{-1}$ | $D^0 \rightarrow K_S \pi^+ \pi^-$ | $A_{CP} = (-0.05 \pm 0.57 \pm 0.54)$% | PRD86.032007 |
Many quasi 2-body intermediate states.

\[ D^0 \rightarrow K_S^+ \pi^- \] decays:

- CF \( D^0 \rightarrow K^{*-} \pi^+; \)
- DCS \( D^0 \rightarrow K^{*-} \pi^+; \)
- CP \( D^0 \rightarrow \rho K_S; \)

Method: **Time-dependent Dalitz Analysis (TDDA)**
Time-dependent Dalitz Analysis

- Many quasi 2-body intermediate states.
  e.g. in $D^0 \rightarrow K_S\pi^+\pi^-$ decays:
  - CF $D^0 \rightarrow K^{*-}\pi^+$;
  - DCS $D^0 \rightarrow K^{*-}\pi^+$;
  - CP $D^0 \rightarrow \rho K_S$;
- Method: Time-dependent Dalitz Analysis (TDDA)
- Dalitz variables: $m_{\pm}^2 = m_{K_SK}^2$
- Time-dependent decay amplitude depends on $m_{\pm}^2$:
  $\mathcal{M}(m_{-}^2, m_{+}^2, t) = \langle K_S\pi^+\pi^- | D^0(t) \rangle$
  $\quad = \frac{1}{2} \mathcal{A}(m_{-}^2, m_{+}^2) [e^{-i\lambda_1 t} + e^{-i\lambda_2 t}] + \frac{1}{2} \overline{\mathcal{A}}(m_{-}^2, m_{+}^2) [e^{-i\lambda_1 t} - e^{-i\lambda_2 t}]$
- Instantaneous amplitude:
  $\mathcal{A}(m_{-}^2, m_{+}^2) = \sum a_r e^{i\phi_r} A_r(m_{-}^2, m_{+}^2) + a_{NR} e^{i\phi_{NR}}$
**BaBar and Belle previous results**

**BaBar@469 fb$^{-1}$ [PRL105,081803]**

\[ D^0 \rightarrow K_S^0 \pi^+ \pi^- \text{ and } D^0 \rightarrow K_S^0 K^+ K^- \]

68%, 95% and 99.9% confidence level contours

Conserved \( CP \) symmetry \((|q/p| = 1 & \phi = 0)\)

\[
\begin{align*}
    x &= (0.16 \pm 0.23 \pm 0.14) \% \\
y &= (0.57 \pm 0.20 \pm 0.15) \% 
\end{align*}
\]

Most accurate determinations of \( x \) and \( y \!\!\!\]

1.9 standard deviation from no-mixing hypothesis

**Belle@540 fb$^{-1}$ [PRL99,131803]**

\[ D^0 \rightarrow K_S^0 \pi^+ \pi^- \]

95% confidence level contours

Conserved \( CP \) symmetry \((|q/p| = 1 & \phi = 0)\)

\[
\begin{align*}
    x &= (0.80 \pm 0.29^{+0.13}_{-0.16}) \% \\
y &= (0.33 \pm 0.24^{+0.10}_{-0.14}) \% 
\end{align*}
\]

CPV allowed \((|q/p| & \phi \text{ free parameters of the fit})\)

\[
\begin{align*}
    |q/p| &= 0.86 \pm 0.30 \pm 0.09 \\
    \phi &= -0.24 \pm 0.30 \pm 0.09 
\end{align*}
\]

2.2 standard deviation Consistent with no CPV!
Assuming no CPV, the World Average in $D^0 \rightarrow K_S h h$ (where $h = K$ or $\pi$) measurements:

\[ x = (0.419 \pm 0.211)\%, \ C.L. = 0.30 \]

\[ y = (0.456 \pm 0.186)\%, \ C.L. = 0.63 \]
$A_{CP}$ in $D^0 \rightarrow K_S \pi^+ \pi^-$

PRD86.032007(2012)

- CDF II data (6.0 $fb^{-1}$), Tevatron $p \bar{p}$ collision, \( \sqrt{s} = 1.96 TeV \).
- $D^0$ from $D^{*+}(2010) \rightarrow D^0 \pi^+$ and $\bar{D}^0$ from its conjugated decay, where the flavor of the charm meson is determined by the charge of the $\pi$.
- Signal and background: artificial neural network with NeuroBayes package

$M_{K_S \pi^+ \pi^-}$, $\Delta M$ fit:
Dalitz-plot fit

\[
FF_r = \frac{\int |a_r e^{i\delta_r} A_r|^2 dM^2_{K_S\pi^{\pm}(RS)} dM^2_{\pi^+\pi^-}}{\int |\sum_j a_j e^{i\delta_j} A_j|^2 dM^2_{K_S\pi^{\pm}(RS)} dM^2_{\pi^+\pi^-}}
\]

<table>
<thead>
<tr>
<th>Resonance</th>
<th>(a)</th>
<th>(\delta) [°]</th>
<th>Fit fractions [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K^*(892)^\pm)</td>
<td>1.911 ± 0.012</td>
<td>132.1 ± 0.7</td>
<td>61.80 ± 0.31</td>
</tr>
<tr>
<td>(K_0^*(1430)^\pm)</td>
<td>2.093 ± 0.065</td>
<td>54.2 ± 1.9</td>
<td>6.25 ± 0.25</td>
</tr>
<tr>
<td>(K_2^*(1430)^\pm)</td>
<td>0.986 ± 0.034</td>
<td>308.6 ± 2.1</td>
<td>1.28 ± 0.08</td>
</tr>
<tr>
<td>(K^*(1410)^\pm)</td>
<td>1.092 ± 0.069</td>
<td>155.9 ± 2.8</td>
<td>1.07 ± 0.10</td>
</tr>
<tr>
<td>(\rho(770))</td>
<td>1</td>
<td>0</td>
<td>18.85 ± 0.18</td>
</tr>
<tr>
<td>(\omega(782))</td>
<td>0.038 ± 0.002</td>
<td>107.9 ± 2.3</td>
<td>0.46 ± 0.05</td>
</tr>
<tr>
<td>(f_0(980))</td>
<td>0.476 ± 0.016</td>
<td>182.8 ± 1.3</td>
<td>4.91 ± 0.19</td>
</tr>
<tr>
<td>(f_2(1270))</td>
<td>1.713 ± 0.048</td>
<td>329.9 ± 1.6</td>
<td>1.95 ± 0.10</td>
</tr>
<tr>
<td>(f_0(1370))</td>
<td>0.342 ± 0.021</td>
<td>109.3 ± 3.1</td>
<td>0.57 ± 0.05</td>
</tr>
<tr>
<td>(\rho(1450))</td>
<td>0.709 ± 0.043</td>
<td>8.7 ± 2.7</td>
<td>0.41 ± 0.04</td>
</tr>
<tr>
<td>(f_0(600))</td>
<td>1.134 ± 0.041</td>
<td>201.0 ± 2.9</td>
<td>7.02 ± 0.30</td>
</tr>
<tr>
<td>(\sigma_2)</td>
<td>0.282 ± 0.023</td>
<td>16.2 ± 9.0</td>
<td>0.33 ± 0.04</td>
</tr>
<tr>
<td>(K^*(892)^\pm)(DCS)</td>
<td>0.137 ± 0.007</td>
<td>317.6 ± 2.8</td>
<td>0.32 ± 0.03</td>
</tr>
<tr>
<td>(K_0^*(1430)^\pm)(DCS)</td>
<td>0.439 ± 0.035</td>
<td>156.1 ± 4.9</td>
<td>0.28 ± 0.04</td>
</tr>
<tr>
<td>(K_2^*(1430)^\pm)(DCS)</td>
<td>0.291 ± 0.034</td>
<td>213.5 ± 6.1</td>
<td>0.11 ± 0.03</td>
</tr>
<tr>
<td>Non-Resonant</td>
<td>1.797 ± 0.147</td>
<td>94.0 ± 5.3</td>
<td>1.64 ± 0.27</td>
</tr>
<tr>
<td>Sum</td>
<td>107.25 ± 0.65</td>
<td></td>
<td></td>
</tr>
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A\textsubscript{CP} result

- overall integrated CP asymmetry:

\[
A_{CP} = \frac{\int \frac{|M|^2 - \overline{|M|^2}}{|M|^2 + \overline{|M|^2}} dM^2_{K_S^0 \pi^\pm (RS)} dM^2_{\pi^+ \pi^-}}{\int dM^2_{K_S^0 \pi^\pm (RS)} dM^2_{\pi^+ \pi^-}},
\]

- CPV amplitude \( b_j \); phase \( \phi_j \), matrix elements for \( D^0 \):

\[
\mathcal{M}(\overline{M}) = a_0 e^{i\delta_0} + \sum_j a_j e^{i(\delta_j \pm \phi_j)(\pm \frac{b_j}{a_j})A_j}
\]

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<td>( K^*(892)^{\pm} )</td>
<td>+0.004 ± 0.004 ± 0.011</td>
<td>−0.8 ± 1.4 ± 1.3</td>
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<td>( K^*_7(1430)^{\pm} )</td>
<td>+0.044 ± 0.028 ± 0.041</td>
<td>−1.8 ± 1.7 ± 2.2</td>
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<tr>
<td>( K^*_0(1430)^{\pm} )</td>
<td>+0.018 ± 0.024 ± 0.023</td>
<td>−1.1 ± 1.8 ± 1.1</td>
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<td>( K^*(1410)^{\pm} )</td>
<td>−0.010 ± 0.037 ± 0.021</td>
<td>−1.6 ± 1.9 ± 2.2</td>
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<td>−0.003 ± 0.002 ± 0.000</td>
<td>−1.8 ± 2.2 ± 1.4</td>
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<td>( f_0(980) )</td>
<td>−0.001 ± 0.005 ± 0.004</td>
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<td>( f_2(1270) )</td>
<td>−0.035 ± 0.037 ± 0.013</td>
<td>−2.0 ± 1.9 ± 2.1</td>
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<td>( f_0(1370) )</td>
<td>−0.002 ± 0.008 ± 0.021</td>
<td>−0.1 ± 1.7 ± 2.8</td>
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<tr>
<td>( \rho(1450) )</td>
<td>−0.016 ± 0.022 ± 0.135</td>
<td>−1.7 ± 1.7 ± 3.9</td>
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<tr>
<td>( f_0(600) )</td>
<td>−0.012 ± 0.017 ± 0.025</td>
<td>−0.3 ± 1.5 ± 1.4</td>
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<tr>
<td>( \sigma_2 )</td>
<td>−0.011 ± 0.012 ± 0.004</td>
<td>−0.2 ± 2.9 ± 1.1</td>
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<td>( K^*(892)^{\pm}(DCS) )</td>
<td>+0.001 ± 0.005 ± 0.002</td>
<td>−3.8 ± 2.3 ± 1.2</td>
</tr>
<tr>
<td>( K^*_7(1430)^{\pm}(DCS) )</td>
<td>+0.022 ± 0.024 ± 0.035</td>
<td>−3.3 ± 4.0 ± 3.9</td>
</tr>
<tr>
<td>( K^*_0(1430)^{\pm}(DCS) )</td>
<td>−0.018 ± 0.029 ± 0.017</td>
<td>+4.2 ± 5.3 ± 3.0</td>
</tr>
</tbody>
</table>

For the overall integrated CP-violation asymmetry:

\[ A_{CP} = (-0.05 \pm 0.57 \pm 0.54)\% \]
consistent with SM prediction.
1 Introduction
- $D^0 - \bar{D}^0$ mixing formalism
- Experimental methods
- $D^0$ self-conjugated decays

2 $D^0 \to K_{S}hh$ measurements
- Available charm samples
- All published results’ table
- Mixing and CPV at BaBar and Belle

3 New result in $K_{S}\pi^+\pi^-$ at Belle
- Signal and backgrounds
- Dalitz model and plot
- Systematics estimation
- Final result

4 Summary
(1) **Signal yield and purity**

<table>
<thead>
<tr>
<th>Data</th>
<th>total yield</th>
<th>purity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(4S), \Upsilon(5S)$</td>
<td>921 fb(^{-1})</td>
<td>1.23 (M)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>540 fb(^{-1})</td>
<td>921 fb(^{-1})</td>
<td>1.7</td>
</tr>
<tr>
<td>signal yield</td>
<td>534 (K)</td>
<td>1231 (K)</td>
<td>2.3</td>
</tr>
</tbody>
</table>

(2) **\(M - Q\) distribution:**

![Graphs showing \(M - Q\) distribution](image)
Dalitz Model of $D^0 \rightarrow K_S\pi^+\pi^-$

Three different models used to describe the decay amplitude:

$$A(m_-^2, m_+^2) = B_{r\neq S-wave} + K_{\pi\pi S-wave} + L_{K\pi S-wave}$$

- **P,D wave:** Breit-Wigner model (used 12 resonances)
  $$B_{r\neq S-wave} = \sum_{r\neq S-wave} a_r e^{i\pi r} A_r(m_-^2, m_+^2).$$

- **$\pi^+\pi^-$ S-wave:** K-matrix model
  $$F_1(s) = \sum_j \left[ I - iK(s)\rho(s) \right]^{-1}_j P_j(s).$$

- **$K\pi$ S-wave:** LASS model
  $$T = \sin \delta_B e^{\delta_B} + \sin \delta_R e^{\delta_R} e^{2i\delta_B}.$$  
  $$\tan \delta_R = \frac{m_{R}\Gamma(s)}{m_{R}^2 - m_{K\pi}^2}; \quad \cot \delta_B = \frac{1}{ap} + \frac{rp}{2}.$$
Dalitz plot and proper-time fit

\[ \chi^2/ndf = 1.207 (ndf = 14264 - 42) \]

\[ D^0 \text{ lifetime:} \]
\[ \tau = 410.3 \pm 0.4 \text{ fs} \]
\[ (\tau_{PDG} = 410.1 \pm 1.5 \text{ fs}) \]
preliminary result under no CP-violation:
\[ x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%, \quad y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\% \]

Two kinds of systematics:

(1) experimental sys. uncertainty:

<table>
<thead>
<tr>
<th>Source</th>
<th>((\Delta x) \times 10^{-4})</th>
<th>((\Delta y) \times 10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Candidate selection</td>
<td>+1.05</td>
<td>+1.87</td>
</tr>
<tr>
<td>Signal and backgrounds yields</td>
<td>±0.30</td>
<td>±0.27</td>
</tr>
<tr>
<td>Wrong tagged events’ fraction</td>
<td>−0.67</td>
<td>−0.45</td>
</tr>
<tr>
<td>Time resolution of signal</td>
<td>−1.39</td>
<td>−0.92</td>
</tr>
<tr>
<td>Efficiency</td>
<td>−1.13</td>
<td>−2.09</td>
</tr>
<tr>
<td>Combinatorial’s PDF</td>
<td>+1.90</td>
<td>+2.28</td>
</tr>
<tr>
<td>(K^*(892)) DCS/CF reduced by 5%</td>
<td>−4.82</td>
<td>−3.88</td>
</tr>
<tr>
<td>(K^*_2(1430)) DCS/CF reduced by 5%</td>
<td>+1.71</td>
<td>−0.67</td>
</tr>
<tr>
<td>Total</td>
<td>+2.78</td>
<td>+3.74</td>
</tr>
</tbody>
</table>

(2) Modeling sys. uncertainty:

<table>
<thead>
<tr>
<th>Fit model</th>
<th>((\Delta x) \times 10^{-4})</th>
<th>((\Delta y) \times 10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonances’ M &amp; (\Gamma) error</td>
<td>±1.40</td>
<td>±1.21</td>
</tr>
<tr>
<td>Remove (K^*(1680)^+)</td>
<td>−1.78</td>
<td>−3.02</td>
</tr>
<tr>
<td>Remove (K^*(1410)^\pm)</td>
<td>−1.16</td>
<td>−3.62</td>
</tr>
<tr>
<td>Remove (\rho(1450))</td>
<td>+2.13</td>
<td>+0.30</td>
</tr>
<tr>
<td>Form factors</td>
<td>+4.05</td>
<td>+2.35</td>
</tr>
<tr>
<td>(\Gamma(q^2) = constant)</td>
<td>+3.33</td>
<td>−1.61</td>
</tr>
<tr>
<td>Angular dependence</td>
<td>−8.46</td>
<td>−3.86</td>
</tr>
<tr>
<td>K-matrix formalism</td>
<td>−2.16</td>
<td>+1.79</td>
</tr>
<tr>
<td>Total</td>
<td>+5.83</td>
<td>+3.21</td>
</tr>
</tbody>
</table>

Systematics is consistent with MC estimations.
Fit results (Belle preliminary):

<table>
<thead>
<tr>
<th>Fit case</th>
<th>Parameter</th>
<th>Fit new result</th>
<th>Belle 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>No CPV</td>
<td>$x(%)$</td>
<td>$0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09}$</td>
<td>$0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14}$</td>
</tr>
<tr>
<td></td>
<td>$y(%)$</td>
<td>$0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06}$</td>
<td>$0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08}$</td>
</tr>
<tr>
<td>No dCPV</td>
<td>$</td>
<td>q/p</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>arg $q/p(\degree)$</td>
<td>$-6 \pm 11^{+3+3}_{-3-4}$</td>
<td>$-14^{+16+5+2}_{-18-3-4}$</td>
</tr>
</tbody>
</table>

No dCPV $\Rightarrow$ no direct CP-violation: $\bar{A}_f = A_f$ when $f = \bar{f}$

- $2.5\sigma$ from no-mixing point in $(x,y)$ plane.
- No hint for indirect CPV.
1 Introduction
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3 New result in $K_S \pi^+ \pi^-$ at Belle
- $A_{CP}$ measurement at CDF

4 Summary
Summary

- $D^0 \rightarrow K_S hh$ gives access to mixing parameters and CPV parameters.
- Brief introduction to the published measurements, related world average; Measurement of CP-violation asymmetry in $D^0 \rightarrow K_S \pi^+ \pi^-$ using $6.0 fb^{-1}$ CDF II data:
  
  $$A_{CP} = (-0.05 \pm 0.57 \pm 0.54)\%$$
Summary

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- Brief introduction to the published measurements, related world average; Measurement of CP-violation asymmetry in $D^0 \to K_S \pi^+ \pi^-$ using $6.0 fb^{-1}$ CDF II data:
  \[ A_{CP} = (-0.05 \pm 0.57 \pm 0.54)\% \]

- Updated measurement of $D^0 - \bar{D}^0$ mixing in $D^0 \to K_S \pi^+ \pi^-$ by TDDA, based on $921 fb^{-1}$ data collected by Belle detector.
Summary

- $D^0 \rightarrow K_S hh$ gives access to mixing parameters and CPV parameters.
- Brief introduction to the published measurements, related world average; Measurement of CP-violation asymmetry in $D^0 \rightarrow K_S \pi^+\pi^-$ using $6.0 fb^{-1}$ CDF II data:
  \[ A_{CP} = (-0.05 \pm 0.57 \pm 0.54)\% \]
- Updated measurement of $D^0 - \overline{D}^0$ mixing in $D^0 \rightarrow K_S \pi^+\pi^-$ by TDDA, based on $921 fb^{-1}$ data collected by Belle detector.
  - assuming no CPV: most precise measurements of $x$ and $y$ to date.
  \[ x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%, \ y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\% \]
• $D^0 \rightarrow K_{S}h_{hh}$ gives access to mixing parameters and CPV parameters.

• Brief introduction to the published measurements, related world average; Measurement of CP-violation asymmetry in $D^0 \rightarrow K_S\pi^+\pi^-$ using 6.0$fb^{-1}$ CDF II data:

$$A_{CP} = (-0.05 \pm 0.57 \pm 0.54)\%$$

• Updated measurement of $D^0 - \bar{D}^0$ mixing in $D^0 \rightarrow K_S\pi^+\pi^-$ by TDDA, based on 921$fb^{-1}$ data collected by Belle detector.

  • assuming no CPV: most precise measurements of $x$ and $y$ to date.

  $$x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%, \quad y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\%$$

  • Search for CPV: measurement of CP parameters $|q/p|$ and $\arg(q/p)$ show no hint for indirect CPV.

  $$|q/p| = 0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}, \quad \arg(q/p)(^o) = -6 \pm 11^{+3+3}_{-3-4}.$$
Thank you!
Event reconstruction and selection

Event Reconstruction:

- $K_S$ selection ($K_S \rightarrow \pi^+\pi^-$):
  - Common vertex separated from IR.
  - $|M_{\pi^+\pi^-} - m_{K_S}| < 10\,MeV/c^2$
  - decay vertex resolution:
    - $\pi$ from $D^0$ at least 4 hits on SVD.
    - reconstructed with charged $\pi$ track only.

Selection criteria:

- vertex fit constraint: $\sum \chi^2 < 100$
- proper time error $\sigma_t < 1000\,fs$
Results for $D^0 \to K_S\pi^+\pi^-$ Dalitz-plot parameters obtained from mixing fit, including complex amplitudes, $\pi\pi$ S-wave and $K_S\pi$ S-wave parameters and fractions for each intermediate components. The errors are statistical only.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(892)^-$</td>
<td>1.590 ± 0.003</td>
<td>131.8 ± 0.2</td>
</tr>
<tr>
<td>$K_0^*(1430)^-$</td>
<td>2.059 ± 0.010</td>
<td>-194.6 ± 1.7</td>
</tr>
<tr>
<td>$K^*_2(1430)^-$</td>
<td>1.150 ± 0.009</td>
<td>-41.5 ± 0.4</td>
</tr>
<tr>
<td>$K^*(1410)^-$</td>
<td>0.496 ± 0.011</td>
<td>83.4 ± 0.9</td>
</tr>
<tr>
<td>$K^*(1680)^-$</td>
<td>1.556 ± 0.097</td>
<td>-83.2 ± 1.2</td>
</tr>
<tr>
<td>$K^*(892)^+$</td>
<td>0.139 ± 0.002</td>
<td>-42.1 ± 0.7</td>
</tr>
<tr>
<td>$K^*_0(1430)^+$</td>
<td>0.176 ± 0.007</td>
<td>-102.3 ± 2.1</td>
</tr>
<tr>
<td>$K^*_2(1430)^+$</td>
<td>0.077 ± 0.007</td>
<td>-32.2 ± 4.7</td>
</tr>
<tr>
<td>$K^*(1410)^+$</td>
<td>0.248 ± 0.010</td>
<td>-145.7 ± 2.9</td>
</tr>
<tr>
<td>$K^*(1680)^+$</td>
<td>1.407 ± 0.053</td>
<td>86.1 ± 2.7</td>
</tr>
<tr>
<td>$\rho(770)$</td>
<td>1 (fixed)</td>
<td>0 (fixed)</td>
</tr>
<tr>
<td>$\omega(782)$</td>
<td>0.0370 ± 0.0004</td>
<td>114.9 ± 0.6</td>
</tr>
<tr>
<td>$f_2(1270)$</td>
<td>1.300 ± 0.013</td>
<td>-31.6 ± 0.5</td>
</tr>
<tr>
<td>$\rho(1450)$</td>
<td>0.532 ± 0.027</td>
<td>80.8 ± 2.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\pi\pi$ S-wave Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
</tr>
<tr>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$\beta_3$</td>
</tr>
<tr>
<td>$\beta_4$</td>
</tr>
<tr>
<td>$f_{11}^{\text{prod}}$</td>
</tr>
<tr>
<td>$f_{12}^{\text{prod}}$</td>
</tr>
<tr>
<td>$f_{13}^{\text{prod}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$K_S\pi$ S-wave Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$(MeV/$c^2$)</td>
</tr>
<tr>
<td>$\Gamma$(MeV/$c^2$)</td>
</tr>
<tr>
<td>$F$</td>
</tr>
<tr>
<td>$\phi_F$(rad)</td>
</tr>
<tr>
<td>R</td>
</tr>
<tr>
<td>$\phi_R$(rad)</td>
</tr>
<tr>
<td>$a$(GeV/$c^{-1}$)</td>
</tr>
<tr>
<td>$r$(GeV/$c^{-1}$)</td>
</tr>
<tr>
<td>$K^*(892)$</td>
</tr>
<tr>
<td>$M_{K^*(892)}$(MeV/$c^2$)</td>
</tr>
<tr>
<td>$\Gamma_{K^*(892)}$(MeV/$c^2$)</td>
</tr>
</tbody>
</table>
Unbinned time-dependent Dalitz fit

(1) **Likelihood function**: to fit the Dalitz plot distribution.

\[ 2 \ln \mathcal{L} = 2 \sum_{i=1}^{n} \ln \sum_X f_i^X p_X(m_{+,i}^2, m_{-,i}^2) \]

"X" stand for signal (sig), random slow π (rnd) and combinatorial background (cmb).

(2) **Normalized signal PDF**:

\[
p_{\text{sig}}(m_{+,i}^2, m_{-,i}^2, t_i) = \frac{\int_0^{+\infty} dt' \ R_{\text{sig}}(t_i - t') |M(m_{+,i}^2, m_{-,i}^2, t')|^2 \epsilon(m_{+,i}^2, m_{-,i}^2)}{\int_0^{+\infty} dt \int_D dm_+^2 dm_-^2 |M(m_+^2, m_-^2, t)|^2 \epsilon(m_+^2, m_-^2)},
\]

\[
 f_i^X \quad \text{event-by-event fraction: determined by 2D } M - Q \text{ fit;}
\]

\[
 \epsilon \quad \text{Dalitz plot efficiency function: estimated from MC}
\]

\[
p_{bkg} \quad \text{background PDF: estimated by sideband}
\]

\[
 R_{\text{sig}} \quad \text{signal resolution function.}
\]
Mode optimization in $D^0 \rightarrow K_S K^+ K^-$ at Belle

$D^0 \rightarrow K_S \pi^+ \pi^-$ DP model optimization

<table>
<thead>
<tr>
<th></th>
<th>$P,D$-wave</th>
<th>$\pi \pi$ S-wave</th>
<th>$K \pi$ S-wave</th>
<th>NR</th>
<th>$-2\Delta \ln \zeta$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle(2007)</td>
<td>BW(12Res.)</td>
<td>BW</td>
<td>BW</td>
<td></td>
<td>$a_{NRE}^{k0sr}$</td>
<td>0(default)</td>
</tr>
<tr>
<td>Fit 2</td>
<td>BW(12)</td>
<td>K-matrix(I)</td>
<td>BW</td>
<td>0</td>
<td>620</td>
<td>1.776</td>
</tr>
<tr>
<td>Fit 3</td>
<td>BW(12)</td>
<td>K-matrix(II)</td>
<td>BW</td>
<td>0</td>
<td>261</td>
<td></td>
</tr>
<tr>
<td>Fit 4</td>
<td>BW(12)</td>
<td>BW</td>
<td>LASS</td>
<td>0</td>
<td>-1707</td>
<td></td>
</tr>
<tr>
<td>Fit 5</td>
<td>BW(12)</td>
<td>BW</td>
<td>LASS</td>
<td>$a_{NRE}^{k0sr}$</td>
<td>-2021</td>
<td>1.252</td>
</tr>
<tr>
<td>Babar2010</td>
<td>BW(8)</td>
<td>K-matrix(II)</td>
<td>LASS</td>
<td>0</td>
<td>-1468</td>
<td></td>
</tr>
<tr>
<td>Fit 7</td>
<td>BW(12)</td>
<td>K-matrix(II)</td>
<td>LASS</td>
<td>0</td>
<td>-1886</td>
<td>1.246</td>
</tr>
</tbody>
</table>

➢ For the $\pi \pi$-S wave, BW and K-matrix model tested
➢ For the $K \pi$-S wave, BW and LASS model tested
Gsim MC test in $D^0 \rightarrow K_S K^+ K^-$ at Belle

![Background included MC tests](image)

Background included fit results:

<table>
<thead>
<tr>
<th>Input x,y(%)</th>
<th>Mean of outputs(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>(0.023±0.078, 0.049±0.063)</td>
</tr>
<tr>
<td>(1,1)</td>
<td>(1.014±0.080, 0.925±0.068)</td>
</tr>
</tbody>
</table>

$x, y$ outputs consistent with true value.
**Measurement of lifetime difference between CP-even and CP-odd eigenstates**

\[
\sqrt{s_0} = m_{K^+K^-} \text{ dependent CP mixture}
\]

\(\leftrightarrow\) ON region: mainly CP-odd \((\phi(1020)K_S^0)\)

\(\leftrightarrow\) OFF region: mainly CP-even \((a_0(980)^0K_S^0)\)

\[
d^2N(s_0,t) = a_1(s_0)e^{-(1+y_{CP})t/\tau_{D^0}} + a_2(s_0)e^{-(1-y_{CP})t/\tau_{D^0}}
\]

\(f_{ON}, f_{OFF}\) are CP-even fractions in ON and OFF regions, determined using 8-resonant Dalitz model (REF!!!)

\[
y_{CP} = + (0.11 \pm 0.61\text{(stat.)} \pm 0.52\text{(syst.)})\%
\]
mixing par. in $D^0 \rightarrow K_S \pi^+ \pi^- K_S K^+ K^-$ at BaBar

TABLE I. Results from the mixing fits. The first uncertainty is statistical, the second systematic and the third systematic from the amplitude model. For the nominal fit, the corresponding correlation coefficients between $x$ and $y$ are 3.5%, 16.0%, and -2.7%, respectively.

<table>
<thead>
<tr>
<th>Fit type</th>
<th>$x/10^{-3}$ (1 σ)</th>
<th>$y/10^{-3}$ (1 σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>$1.6 \pm 2.3 \pm 1.2 \pm 0.8$</td>
<td>$5.7 \pm 2.0 \pm 1.3 \pm 0.7$</td>
</tr>
<tr>
<td>$K_S^{0} \pi^+ \pi^-$</td>
<td>$2.6 \pm 2.4$</td>
<td>$6.0 \pm 2.1$</td>
</tr>
<tr>
<td>$K_S^{0} K^+ K^-$</td>
<td>$-13.6 \pm 9.2$</td>
<td>$4.4 \pm 5.7$</td>
</tr>
<tr>
<td>$D^0$</td>
<td>$0.0 \pm 3.3$</td>
<td>$5.5 \pm 2.8$</td>
</tr>
<tr>
<td>$\tilde{D}^0$</td>
<td>$3.3 \pm 3.3$</td>
<td>$5.9 \pm 2.8$</td>
</tr>
</tbody>
</table>

FIG. 3. Central value (point) and C.L. contours (including statistical, systematic, and amplitude model uncertainties) in the $x$-$y$ plane for C.L. = 68.3%, 95.4%, 99.7%. The no-mixing point is shown as a plus sign (+).