Partial branching fractions and extraction of $|V_{ub}|$ from inclusive semileptonic B decays

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**|V_{ub}| Determination**

Limiting factor in CKM precision tests; known much less well than |V_{cb}|

CKM suppressed $V_{ub} \sim 0.1 \times V_{cb}$ - therefore harder to measure

$V_{ub}$: @8% precision, dominated by theoretical uncertainties

\[
\Gamma(b \rightarrow u\ell\bar{\nu}) = \frac{G_F^2}{192\pi^2} |V_{ub}|^2 m_b^5 \left(1 + \text{補正項} \right)
\]

The problem: $b \rightarrow c\ell\nu$ decay

\[
\frac{\Gamma(b \rightarrow u\ell\bar{\nu})}{\Gamma(b \rightarrow c\ell\bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}
\]

How can we suppress 50× larger background?
How can we reach 4% precision on |V_{ub}|?
**|V_{ub}| Measurement**

- Cut away $b \to c l v$ Lose a part of the $b \to u l v$ signal

  - We measure
    
    \[ \Gamma(B \to X_u \ell \nu) \times f_C = |V_{ub}|^2 \zeta_C \]

  - Total $b \to u l v$ rate
  - Fraction of the signal that pass the cut
  - Requires the knowledge of the $b$ quark's motion inside the $B$ meson

- Must be corrected for QCD

  \[ \Gamma(B \to X_u \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left[ 1 - O\left(\frac{\alpha_s}{\pi}\right) - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \cdots \right] \]

  - Main uncertainty ($\pm 5\%$) from $m_b^5 \to \pm 2.5\%$ on $|V_{ub}|$, correlated between all measurements/experiments!

- But we need the reasonable fraction (e.g., $E_\ell > 2$ GeV) of the rate.
Detecting $b \rightarrow u \nu$

**Inclusive:** Use $m_u << m_c$ difference in kinematics

- Maximum lepton energy 2.64 vs. 2.31 GeV
- First observations (1990) used this technique
- Only 6% of signal accessible

- Small theory error requires low $E_{\text{lepton}}$ cut
- Small BG uncertainty requires high $E_{\text{lepton}}$ cut
- Measure partial BR in a region where S/N is acceptable and that the width is reliably calculable.

- Many techniques used:
  - Endpoint, Improved Endpoint, Simulated Annealing, SF factorisation, Full Reconstruction tagging etc.
Endpoint method

Subtract offpeak data scaled to onpeak luminosity bin-by-bin.

Fit MC to data in low energy region to constrain $B\to Xc l v$ from data.

$B\to X_u l v$, $B\to D_{lv}+B\to D^{*}_{lv}$ (ratio fixed)

$B\to D^{**}_{lv}, B\to D^{(*)}_{llv}$ (Goity & Roberts)

$B\to X_u l v$, $B\to D_{lv}+B\to D^{*}_{lv}$ (D/D* fixed)

$B\to D^{**}_{lv} D^{*}/D+D^{*}$ fitted ($(D_1+D_2)/D^{*}$ fixed)

| Source | Luminosity | $\Delta BR \times 10^{-4}$ | $|V_{ub}| \times 10^{-3}$ |
|--------|------------|-----------------------------|---------------------------|
| (PRL 88, 231803, 2002) | Belle $27fb^{-1}$: $E_e > 1.9 GeV$ | $8.47 \pm 0.37^{stat} \pm 1.53^{sys}$ | $4.61(\pm 9.3 + 5.0 - 6.7)$ |
| (PLB621, 28, 2005) | BaBar: $E_e > 2.0 GeV$ | $5.72 \pm 0.41^{(stat)} \pm 0.65^{(syst)}$ | $4.13(\pm 5.6 + 5.6 - 8.2)$ |
| (PRD 73, 012006, 2006) | CLEO: $E_e > 2.0 GeV$ | $4.22 \pm 0.33^{stat} \pm 1.78^{sys}$ | $3.77(\pm 12. + 6.9 - 10.)$ |

Simultaneous fit for non-$B\bar{B}$, $B\to X_u l v$, $B\to D_{lv}$

$B\to D^{*}_{lv}$, $B\to D^{**}_{lv}$, $B\to D^{(*)}_{llv}$, other BG

$\sim 25 \%$ acceptance
Improved Endpoint method

Maximally allowed hadronic mass for given \( E_l \) & \( q^2 \) can be determined:

\[
\begin{align*}
  s_h^\text{max} &= m_B^2 + q^2 - 2m_B (E_l \eta_+ + q^2 \eta_+/4E_e), & \pm 2E_e &> \pm \sqrt{q^2} \eta_+ & \eta_\pm &= \sqrt{(1 \pm \beta)/(1 \mp \beta)} \\
  s_h^\text{max} &= m_B^2 + q^2 - 2m_B \sqrt{q^2}, & \text{otherwise}
\end{align*}
\]

BaBar (PRL 95, 111801, 2005

**Principal ingredients (BABAR):**

1) \( p_\nu = p_\text{miss} = (E_\text{miss}, \vec{p}_\text{miss}) \rightarrow (|\vec{p}_\text{miss}|, \vec{p}_\text{miss}) \)

2) * track & neutral selection
   * Only one high energetic lepton
   * \(|\vec{p}_\text{miss}|, E_\text{miss} - |\vec{p}_\text{miss}|, \cos \theta_\text{miss}, \cos \theta_\text{thrust}, e_\)

3) Signal region: \( E_l^* \) & \( s_h^\text{max} \Rightarrow B \rightarrow X_c \nu \) suppression

\( \text{core } |\vec{p}_\text{miss}|/q^2 \text{- resolution: 0.32 GeV/2.5 GeV}^2 \)
   * dominated by particle losses
   * complicated by \( K_L \), split-offs, etc.
## Summary of Endpoint methods

### ΔBR Errors in %

<table>
<thead>
<tr>
<th>Method</th>
<th>El cut</th>
<th>Integrated Luminosities (fb-1)</th>
<th>Uncertainties on BF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>onpeak</td>
<td>offpeak</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Stat</td>
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<tr>
<td>El</td>
<td></td>
<td></td>
<td>CLEO (PRL 88, 231803, 2002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Belle (PLB621, 28, 2005)</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BaBar (PRD 73, 012006, 2006)</td>
<td>2.1</td>
</tr>
<tr>
<td>El-q2</td>
<td></td>
<td>BaBar (PRL 95, 111801, 2005 PRL 97, 019903 (2006) Err.)</td>
<td>2.0</td>
</tr>
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</table>

### Systematics

<table>
<thead>
<tr>
<th>Systematics</th>
<th>Select</th>
<th>non-BB</th>
<th>FSR/Brem</th>
<th>KL</th>
<th>Neutral</th>
<th>B → Xu l ν</th>
<th>D-&gt; e, mu</th>
<th>D-&gt;KL</th>
<th>FF</th>
<th>Excl BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>El</td>
<td>6.8</td>
<td>2.4</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
<td>1.1</td>
<td>&lt;0.5</td>
<td>-</td>
<td>2.4</td>
<td>2.8</td>
</tr>
<tr>
<td>El-q2</td>
<td>-</td>
<td>-</td>
<td>3.8</td>
<td>2.0</td>
<td>3.4</td>
<td>4.4</td>
<td>2.1</td>
<td>4.5</td>
<td>0.3</td>
<td>2.0</td>
</tr>
</tbody>
</table>

- Current data sets from Belle and BaBar have low offpeak/onpeak ratios.
- Prospect for stat. error <2.5% by relaxing continuum suppression techniques.
- B→Xc l ν systematics have improved significantly over 5 years.
Best access to kinematics with $B$ meson “beam” method

Not to scale!

$B_{\text{reco}} \rightarrow D(\ast)\gamma$

$B_{\text{sig}} \rightarrow X\ell\nu$

high purity, low efficiency

$E_i = $ lepton energy (in $B$ rest frame)

$q^2 = $ lepton-neutrino mass squared

$m_X = $ hadron system mass (all remaining particles)

$P^+ = E_X - |P_X| (P_X = p_{\text{beam}} - p_{B\text{tag}} - p_{\ell} - p_{\nu})$

Signal events have smaller $M_X$ and $P^+ \rightarrow$ Larger $E_i$ and $q^2$
Measurement technique

1. Signal side
Reconstruct high momentum lepton
$p_{\text{lepton}}^{\text{CMS}} > 1 \text{ GeV/c}$
No strangeness
$N(K^\pm, K^0_S) = 0$ (KL cut used by Belle 2005)

2. Event Level
$\Sigma Q=0,$
$Q(B_{\text{reco}+}) \times Q(\text{lep}) = -1$
$M_{\text{miss}}, \text{etc.}$

3. Fit $M_{bc}$ in bins of $P_+$, $M_x$, $q^2$

4. Fit to $P_+$, $M_x$, $q^2$ with various background and signal floated to determine background yield.

5. Measure relative BR to control systematics
Belle tagged method

PRL .95:241801 (2005)

253 fb$^{-1}$

| $|V_{ub}| \times 10^{-3}$ | BLNP | $\Delta |V_{ub}|$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_X &lt; 1.7$</td>
<td>$4.70(\pm 5.1\pm 6.0\pm 4.2\pm 5.1)$</td>
<td>10%</td>
</tr>
<tr>
<td>$m_X &lt; 1.7, q^2 &gt; 8$</td>
<td>$4.09(\pm 4.6\pm 4.9\pm 4.4\pm 3.7)$</td>
<td>9%</td>
</tr>
<tr>
<td>$P+&lt;0.66$</td>
<td>$4.19(\pm 4.8\pm 7.2\pm 5.7\pm 3.3)$</td>
<td>11%</td>
</tr>
</tbody>
</table>

$\sim 40-60 \%$ acceptance

CKM Workshop, Rome Sept 2008

Phillip Urquijo
### Babar tagged method

**PRL 100, 171802 (2008)**

- $B \to X_u l n$
- $B \to X_c l n$ (bkgd)
- $B \to X_u l n$ (outside selected region)

Background subtracted

![Graphs of $M_X$ and $P_+$ distributions for different kinematic regions.](image)

| Kinematic Region                  | $B(B \to X_u l v) \times 10^{-3}$ | $|V_{ub}| (10^{-3})$ | Theory |
|----------------------------------|---------------------------------|---------------------|--------|
|                                 | $\Delta(\text{stat. sys. th.})$ | $\%\Delta(\text{stat. sys. th.})$ |        |
| $M_X < 1.55 \text{ GeV}/c^2$    | $1.18 \pm 0.09 \pm 0.07 \pm 0.01$ | $4.27 (\pm 3.7 \pm 3.0 \pm 7.0)$ | BLNP   |
|                                 |                                | $4.56 (\pm 3.7 \pm 3.0 \pm 7.0)$ | DGE    |
| $P_+ < 0.66 \text{ GeV}/c^2$    | $0.95 \pm 0.10 \pm 0.08 \pm 0.01$ | $3.88 (\pm 4.9 \pm 4.1 \pm 7.2)$ | BLNP   |
|                                 |                                | $3.99 (\pm 4.9 \pm 4.1 \pm 6.0)$ | DGE    |
| $M_X < 1.7 \text{ GeV}/c^2 \& q^2 > 8.0 \text{ GeV}^2/c^2$ | $0.81 \pm 0.08 \pm 0.07 \pm 0.02$ | $4.57 (\pm 5.0 \pm 4.2 \pm 4.6)$ | BLNP   |
|                                 |                                | $4.64 (\pm 5.0 \pm 4.2 \pm 5.4)$ | DGE    |
|                                 |                                | $4.93 (\pm 5.0 \pm 4.2 \pm 7.3)$ | BLL     |

~40-60 % acceptance

CKM Workshop, Rome Sept 2008

 Phillip Urquijo
Extension: Hadronic mass Moments

Measure hadronic mass spectrum over full $M_x$ range
Mass moments related to $mb$, with $m_x^2 < 6.4 \text{ GeV}^2$

Calculations of Gambino, Ossola, Uraltsev
JHEP09(2005)010

First measurement of $m_b$ in $B \rightarrow X_u \ell \nu$ decays
(in the Kinetic scheme)

$m_b = \ 4.604 \pm 0.250 \text{ GeV}$
compatible with Global Fit
The irreducible uncertainties in the measurements to date are related to limited phase space.

Solution: exploit the many non-linear correlations between kinematic and event variables available in B-beam sample that separate $b \rightarrow u$ and $b \rightarrow c$.

Boosted decision tree based selection, use ~20 event parameters from the full reconstruction sample.

No need to place stringent, hard cuts that result in zero efficiency!

1. Signal side: Reconstruct high momentum lepton: $p_{lep}^{B^*} > 1$ GeV/c

2. Event Level: $Q(B_{reco}^+) \times Q(lep) = -1$

3. BDT cut with many input par's: $M_{miss}^2$, $dZ$, $dr$, $Q_{total}$, $Q_{lepton}$, $N_{lepton}$, $Q(B)$, $D^*$ partial reco etc.....

4. Combinatorial estimated from MC, normalisation from sideband region. (same approach as $V_{cb}$ moments analyses)

5. 2D fit to $M_x$, $q^2$ with backgrounds and signal floated to determine background yield.

6. Measure absolute rate.
Belle Multivariate analysis (NEW @ CKM2008) 2/2

2D fit in q2 mx. (projections shown)

| Kinematic Region | $B(B \rightarrow X_u l \nu) \times 10^{-3}$ | $m_b$ | $|V_{ub}|$ (10^{-3}) % error | Theory          |
|------------------|-----------------------------------------|-------|----------------------------|----------------|
| $P_{lepton} > 1.0$ GeV | $1.96 \times (1 \pm 0.088 \pm 0.076)$ | (kinetic) 4.613 GeV, mupi=0.440 GeV² | 4.42 (± 3.1 ± 5.1) | GGOU (thanks P. Giordano) |
|                  |                                         | (MSbar) 4.243 GeV | 4.47 (± 6.7) | DGE (thanks E. Gardi) |

~90% total phase space, thus theory error less correlated to other Vub determinations
## Tagged method Uncertainty Breakdown

### $\Delta BR$ Errors in %

<table>
<thead>
<tr>
<th></th>
<th>stat</th>
<th>sys.</th>
<th>MC stat</th>
<th>detector/other</th>
<th>$B \rightarrow Xu \ell \nu$</th>
<th>$B \rightarrow cl \ell \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P &gt; 1.0$ GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>BDT</td>
<td></td>
</tr>
<tr>
<td><strong>PRL 100, 171802 (2008)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.8</td>
<td>7.6</td>
</tr>
<tr>
<td>$Mx/q^2$</td>
<td>8.0</td>
<td>7</td>
<td>4.3</td>
<td>3.8</td>
<td>5.2</td>
<td>2.4</td>
</tr>
<tr>
<td>$Mx$</td>
<td>9.0</td>
<td>7</td>
<td>3.2</td>
<td>1.9</td>
<td>3.7</td>
<td>0.9</td>
</tr>
<tr>
<td>$P+$</td>
<td>10.0</td>
<td>8</td>
<td>4.6</td>
<td>3.9</td>
<td>4.0</td>
<td>1.3</td>
</tr>
<tr>
<td>$Mx/q^2$</td>
<td>10.0</td>
<td>8.9</td>
<td>8.9</td>
<td>6.2</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>$Mx$</td>
<td>9.1</td>
<td>7.1</td>
<td>7.1</td>
<td>6.1</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>$P+$</td>
<td>9.4</td>
<td>9.3</td>
<td>9.3</td>
<td>6.4</td>
<td>8.7</td>
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<tr>
<td><strong>Belle Preliminary CKM2008</strong></td>
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<td>BDT</td>
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</tr>
<tr>
<td>$BDT$</td>
<td>8.8</td>
<td>7.6</td>
<td>4.8</td>
<td>3.6</td>
<td>4.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

- Detector effects are better understood and no longer dominate.
- Selection often involves complex fitting techniques ($m_{ES}$)
  - more data improves fit convergence in $m_{ES}$ method.
  - more data will mean more detail in $q^2/Mx/P+$ to account for!
- Dominant systematics are in the modelling of signal and background contributions
  - $b \rightarrow u \ell \nu$ (resonances, non-resonant contributions)
  - $b \rightarrow c \ell \nu$ ($D$, $D^*$, $D^{**}$ very small)
  - Continuum error large in $q^2/Mx/P+$ analyses.

KL cut powerful (used in Belle $Mx/P+/q^2$ analysis) but modelling/understanding needs huge improvements.
Weak Annihilation

- Small contribution to $B \to Xu \ell \nu$ decays
- Introduces difference between $B^0$ and $B^+$ decays.
- Tag with partial reconstructed $B^0 \to D^{*+} \ell \nu$
- Neutrino mass from $m_\nu^2 = (P_B - P_{D^*-})^2$
- Compare $B^0$ partial rate to $B^+$ averaged rate over large $P_l$ region (to enhance the WA contribution) PRD73, 012006 (2006)

\[
\frac{|\Gamma_{WA}|}{\Gamma_u} < 7.4\% \text{ at } 90\% C.L.

CLEO, studing the $q^2$ spectra PRL96,121801 (2006)

<table>
<thead>
<tr>
<th>$\Delta p$</th>
<th>$\Delta B(B) \cdot 10^4$</th>
<th>$\Delta B(B^0) \cdot 10^4$</th>
<th>$A^{+,0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2 - 2.6 GeV/c</td>
<td>2.31±0.10±0.18</td>
<td>2.62±0.33±0.16</td>
<td>-0.17±0.15±0.11</td>
</tr>
<tr>
<td>2.3 - 2.6 GeV/c</td>
<td>1.46±0.06±0.10</td>
<td>1.30±0.21±0.07</td>
<td>0.08±0.15±0.08</td>
</tr>
<tr>
<td>2.4 - 2.6 GeV/c</td>
<td>0.75±0.04±0.06</td>
<td>0.76±0.15±0.05</td>
<td>-0.05±0.20±0.10</td>
</tr>
</tbody>
</table>

\[
\frac{|\Gamma_{WA}|}{\Gamma_u} < \frac{3.8\%}{f_{WA}(2.3 - 2.6)} \text{ at } 90\% C.L.

arXiv: 0708.1753 383 M BB
Shape Function

Theory doesn’t work everywhere in the phase space → re-sum turn non-perturbative terms

We can determine mean and width of shape function.

- Fit the $b \rightarrow s\gamma$ spectrum to constrain the shape function
- $b$-quark: mass and kinetic energy from $b \rightarrow clv$ and $b \rightarrow s\gamma$ decays
- Plug the SF into the $b \rightarrow ulv$ spectrum calculations
- Ready to extract $|V_{ub}|$

Theory calculation for SF $\geq 2$-loop.

Shape Function needs to be determined from experimental data. alternatives: cut out SF region, or factorise out the SF
SF independent analyses

Exploit the assumption that the leading shape functions are the same for all $b \to q$ transitions.

\[ \int_{0}^{m_{\text{max}}} d\Gamma (b \to u) \frac{d\Gamma (b \to s\gamma)}{d\Gamma (b \to s\gamma)} dE_{\gamma} \]

![Graph showing calculation comparison between LLR and LNP](image)

Golubev, Skovpen, Luth PRD76, 114003 (2007)

Use lepton endpoint spectrum (BaBar)

Uncertainties due to non-perturbative power corrections increase rapidly near the kinematic end point (not included in LLR).

The results agree with analyses that rely on an assumed shape function form.

Measure $Mx$ in Breco sample (89 MBB)

\[ |V_{ub}| = (4.43 \pm 0.30_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.29_{\text{thoo}}) \times 10^{-3} \]
Inclusive $|V_{ub}|$

$m_b(\text{kin}) = 4.601 \pm 0.034 \text{ GeV} \quad \text{HFAG}$

$m_b \text{ (MSbar)} = 4.243 \pm 0.042 \text{ GeV} \quad \text{HFAG}$

**CLEO ($E_u$)**
- $3.77 \pm 0.44 + 0.26 - 0.39$

**BELLE sim. ann. ($m_x$, $q^2$)**
- $4.23 \pm 0.45 + 0.34 - 0.35$

**BELLE ($E_u$)**
- $4.61 \pm 0.43 + 0.23 - 0.31$

**BABAR ($E_u$)**
- $4.13 \pm 0.23 + 0.23 - 0.34$

**BELLE ($m_x$)**
- $3.93 \pm 0.26 + 0.19 - 0.22$

**BABAR $m_x$**
- $4.07 \pm 0.20 + 0.27 - 0.29$

**BABAR $m_x$, $q^2$**
- $4.29 \pm 0.28 + 0.34 - 0.36$

**BABAR $P^+$**
- $3.52 \pm 0.23 + 0.30 - 0.31$

Average +/- exp + theory - theory
- $3.96 \pm 0.15 + 0.20 - 0.23$

**GGOU**

**DGE**

**belle BDT prelim.**

no systematic difference between tagged and untagged results
## Inclusive $|V_{ub}|$ Error Breakdown

<table>
<thead>
<tr>
<th>Source</th>
<th>BLNP</th>
<th>DGE</th>
<th>GGOU</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+8.2 -7.2</td>
<td>+5.7 -4.7</td>
<td>+6.3 - 7.0</td>
</tr>
<tr>
<td><strong>Statistical</strong></td>
<td>2.1</td>
<td>2.2</td>
<td>2.2</td>
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<tr>
<td><strong>Experimental Systematics</strong></td>
<td>2.3</td>
<td>2.1</td>
<td>2.2</td>
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<tr>
<td><strong>b-&gt;c l v model</strong></td>
<td>1.3</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>b-&gt;u l v model</strong></td>
<td>1.4</td>
<td>1.3</td>
<td>1.5</td>
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<tr>
<td><strong>HQ parameters/non. pert</strong></td>
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<tr>
<td><strong>SF + Sub. SF</strong></td>
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<tr>
<td><strong>matching</strong></td>
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<tr>
<td><strong>Weak Annihilation</strong></td>
<td>1.6</td>
<td>1.5</td>
<td>+0.0 -3.1</td>
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<tr>
<td><strong>DGE theory</strong></td>
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<td>+1.2 -0.5</td>
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<tr>
<td><strong>$R_{cut}$</strong></td>
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<td>+0.7 -0.3</td>
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<tr>
<td><strong>q2 tail model</strong></td>
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<tr>
<td><strong>Higher order parameters</strong></td>
<td></td>
<td>+1.3 -0.6</td>
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</tr>
</tbody>
</table>

### what can improve?

- **More data** ($ab^{-1}$) for better background suppression, more off-res in endpoint.
- **$B$ Beam**
- Reduced significantly in tagged meas.
- Exclusive $B \rightarrow \rho/\omega/\eta$ lν recently dominate

### HQE/|$V_{cb}$| fits/ $Y(1S)$ (hit theory and expt. wall!)

- Most theory uncertainties can be reduced significantly if we increase the measured phase space.
- The power dependence of $m_b$ and related theoretical uncertainties scale inversely with phase space coverage.
Comparison of $|V_{ub}|$ extraction

- OPE based + SF
  - **BLNP**: HQE with systematic introduction of SF
    \( \text{PRD71:073006 (2005)} \)
  - **BLL**: phase space in \( m_x-q^2 \) with reduced SF
    \( \text{PRD64:113004 (2001)} \)
  - **GGOU**: kinetic scheme
    \( \text{JHEP 10(2007) 058} \)
  - **LNP (LLR)**: \( b \rightarrow s \gamma \) directly related to \( b \rightarrow u \ell \nu \)
    \( \text{JHEP 0510:084 (PLB 486:86)} \)

No SF introduced \( \Rightarrow \) model non-perturbative QCD

- **DGE**: Dressed Gluon Exponentiation
  \( \text{JHEP 0601:097 (2006)} \)
- **ADFR**: Analytic coupling
  \( \text{PRD74:03400(4,5,6) (2006)} \)

### HFAG Averages

- \( \text{HFAG Ave. (BLNP)} \)
  \( 4.32 \pm 0.16 + 0.32 - 0.27 \)
- \( \text{HFAG Ave. (DGE)} \)
  \( 4.26 \pm 0.14 + 0.19 - 0.13 \)
- \( \text{HFAG Ave. (GGOU)} \)
  \( 3.98 \pm 0.15 + 0.20 - 0.23 \)
- \( \text{HFAG Ave. (ADFR)} \)
  \( 3.76 \pm 0.13 \pm 0.22 \)
- \( \text{HFAG Ave. (BLL)} \)
  \( 4.87 \pm 0.24 \pm 0.38 \)

### BABAR Results

- **BABAR (LLR)**
  \( 4.92 \pm 0.32 \pm 0.36 \)
- **BABAR endpoint (LLR)**
  \( 4.28 \pm 0.29 \pm 0.48 \)
- **BABAR endpoint (LNP)**
  \( 4.40 \pm 0.30 \pm 0.47 \)

\[ |V_{ub}| \left[ \times 10^{-3} \right] \]
How Things Mesh Together

AKA: M. Morii’s HQE plumbing diagram

- $b \rightarrow s \gamma$
- $E_\gamma$
- Inclusive $b \rightarrow ulv$
  - $E_1$
  - $q^2$
  - $m_X$
- $m_b$
- Shape Function
- SSFs
- HQE Fit
- $|V_{ub}|$
- $|V_{cb}|$
- WA
- duality
- Inclusive $b \rightarrow clv$
  - $E_1$
  - $m_X$

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Phillip Urquijo

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Conclusions

**Untagged methods**

**Endpoint:**
Goal: lower lepton energy cut to reduce theoretical uncertainty
Will require improved knowledge of charmed semileptonic background.
Stat. error will be relatively small if we have enough off-resonance data.

**Improved Endpoint:**
Prospects for reducing systematics, phase space cuts can be relaxed
Gain from relaxing $S_h^{\text{max}}$, however, there is fast BG increase

**Tagged Methods**

**Full Reconstruction:**
Better knowledge of exclusive states crucial.
Data has become quite sensitive to the underlying hybrid model MC (though HQ parameters have become better known with charmed semileptonic decays).
Novel BDT approach by Belle promises to reduce phase space dependent uncertainties.

**Overall**
Novel approaches eliminating SF dependence agree with SF, and QCD Model results.
WA tied down.
Inclusive $V_{ub} \sim 8\%$ error shared between theoretical and experimental. Static in the past 2 years.
Why? $|V_{ub}|$ theory errors are highly correlated.