Study of fragmentation functions in $e^+ e^-$ annihilation process at Belle

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on behalf of the Belle Collaboration
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- Summary and conclusions
Motivation for FF studies

How do quasi-free partons fragment into confined hadrons?

- Does spin play a role? Flavour dependence?
- What about transverse momentum distribution (TMD) and its evolution?

- Fragmentation functions (FF's) describe the hadronisation phenomenon: $q/g \rightarrow h$

  $D_q^h(z, Q^2)$

  $z$: fractional momentum of the parent quark carried by the hadron

- FF's are needed in the global analysis of the nucleon structure, e.g. the spin structure (from semi-inclusive Deep-Inelastic-Scattering (SIDIS) and $pp$ collisions)

- FF's are intrinsically linked to QCD confinement → provide access to non-perturbative QCD quantities

- FF's can not be computed on the lattice due to the non-inclusive final states
In $e^+e^-$ annihilation:

\[ Q = \sqrt{s} \]
\[ z = \frac{2E_h}{Q} \approx \frac{E_h}{E_q} \]

- $e^+e^-$ cleanest way to access FF's
- B factories (Belle, BaBar)
  - close in energy to SIDIS (100 GeV$^2$ vs. 2-3 GeV$^2$)
  - Large integrated luminosity, high z reach

CMS energy (mainly):

\[ \sqrt{s} = M_{Y(4S)}c^2 \approx 10.58\text{GeV} \]
Experimental set-up: Belle @ KEKB

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\[ \text{Rate}_i = \text{Lumi.} \times \sigma_i \]

Resonance; $E_{\text{CM}}$(GeV); Integr. lumi.

- $\Upsilon(1S)$: 9.46, 5.75 fb$^{-1}$
- $\Upsilon(2S)$: 10.02, 25 fb$^{-1}$
- $\Upsilon(3S)$: 10.36, 2.95 fb$^{-1}$
- $\Upsilon(4S)$: 10.58, 710.5 fb$^{-1}$
- $\Upsilon(5S)$: 10.87, 121.4 fb$^{-1}$

Off resonance/scan:
- ~100 fb$^{-1}$

\[ \sim 2 \times 10^9 \text{ q\bar{q} pairs} \]

\[ e^+e^- \rightarrow \pi^\pm X \text{ production} \]
Unpolarised FF's from single hadron production

\[ z = \frac{2E_h}{\sqrt{s}}, \quad \sqrt{s} = 10.52 \text{ GeV} \]

- Process studied:
  \[ e^+ e^- \rightarrow h X \]
- Single-hadron cross-sections at leading order (LO) in \( \alpha_s \) is related to the sum of unpolarised fragmentation functions from quark and anti-quark side
  \[ \sigma(e^+ e^- \rightarrow hX) \propto \sum_q e_q^2 \left( D_{1,q}^h(z) + D_{1,q}^h(\bar{z}) \right) \]
- Gluon FF's accessible only through higher-order processes or QCD evolution

\[ \text{LO} \quad F^h(z, s) = \sum_q e_q^2 [D_q^h(z) + D_{q'}^h(\bar{z})] \]
\[ \sum_q e_q^2 \]
\[ \text{NLO} \quad F^h(z, s) = \sum_i \int_{z'}^1 \frac{dz'}{z'} C_i(s; z', \alpha_s) D_q^h(z) \]
Cross-sections for identified $\pi^\pm/K^\pm$

\[ z = \frac{2E_h}{\sqrt{s}}, \quad \sqrt{s} = 10.52 \text{ GeV} \]

Initial/Final State Radiation:
- Exclude events where $E_{\text{CMS}}/2$ changes by more than 0.5%
- Large at low $z$, correction based on MC

\[ \frac{d\sigma_i}{dz} = \frac{1}{L_{\text{tot}}} \epsilon_{\text{joint}}(z) \epsilon^{i}_{\text{ISR/FSR}}(z) S_{zzm}^{-1} \epsilon^{i}_{\text{imp}}(zm) P_{ij}^{-1} N_{ij,raw}(zm) \]

- Smearing Corrections

- Correct for acceptance, $\tau\tau$, $2\gamma$, decay in flight, < 10%

Results used in recent global FF fit (PRD 91, 014035 (2015)):
- Together with other new data substantial improvement in uncertainties
- Good description of B-Factories data

\[ \pi^+ (\text{Statistical, Systematic Uncertainties}) \]
\[ K^+ (\text{Statistical, Systematic Uncertainties}) \]
**Di-hadron fragmentation**

- Single inclusive hadron multiplicities \((e^+e^- \rightarrow hX)\) sum over all available flavours and quarks and antiquarks:

\[
d\sigma(e^+e^- \rightarrow hX)/dz \propto \sum_q e_q^2 (D_{1,q}^h(z, Q^2) + D_{1,q}^\bar{h}(z, Q^2))
\]

- Flavour separation and distinction between quark and antiquark fragmentation, and in particular distinction between favoured (e.g. \(u \rightarrow \pi^+\)) and disfavoured (\(\bar{u} \rightarrow \pi^+\)) fragmentation would be important.

- **Idea:** Use a di-hadron fragmentation, preferably from opposite hemispheres and access favoured and disfavoured combinations, e.g. for \(\pi\pi\):

\[
u\bar{u} \rightarrow \pi^+\pi^- X \propto D_{u,fav}^\pi(z_1, Q^2) \cdot D_{u,fav}^{\bar{\pi}}(z_2, Q^2) + D_{u,dis}^\pi(z_1, Q^2) \cdot D_{u,dis}^{\bar{\pi}}(z_2, Q^2)
\]

\[
u\bar{u} \rightarrow \pi^+\pi^+ X \propto D_{u,fav}^\pi(z_1, Q^2) \cdot D_{u,dis}^{\pi^+}(z_2, Q^2) + D_{u,dis}^{\pi}(z_1, Q^2) \cdot D_{u,fav}^{\pi^+}(z_2, Q^2)
\]

**NB:** This is strictly valid only at LO and for nearly back-to-back hadrons
Cross-sections for a di-hadron production are sensitive to favoured and disfavoured fragmentation depending on charges and hadron types:

- Generally look at 4 x 4 hadron combinations (π⁺, π⁻, K⁺, K⁻) and keep them separated:
  - 6 independent yields

- Use 3 hemisphere combinations (wrt to Thrust axis):
  - same hemisphere (thrust T >0.8)
  - opposite hemisphere (thrust T >0.8 )
  - any combination (no thrust selection)

- Use 16 x 16 (z₁ x z₂) binning within [0.2 , 1] for each z value
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Different. $\sigma$: no hemisphere/thrust cut
Di-hadrons: results for pion pairs

$\pi^+\pi^+$ comparable to $\pi^+\pi^-$ at low $z$, decreasing towards high $z$:

- Favoured and disfavoured fragmentation similar at low $z$
- Disfavoured much smaller at high $z$

$s$ ratio: no hemisphere/thrust cut
MC generator comparison: Low z dominates integral: → Well defined, all tunes agree

High z not well measured, especially at Belle energies: → large spread in tunes

Default Pythia settings and current Belle setting with good agreement

Different. $\sigma$: diagonal $z_1, z_2$ bins only

\begin{align*}
\sigma(z_1, z_2) &= \frac{d\sigma}{dz_1 dz_2} \\
\end{align*}
σ ratio: diagonal $z_1$, $z_2$ bins only

MC generator comparison:
Low $z$ dominates integral:
→ Well defined, all tunes agree

High $z$ not well measured, especially at Belle energies:
→ large spread in tunes

Default Pythia settings and current Belle setting with good agreement
Goal:
Study the correlation between the transverse spin of the quark and the transverse momentum of the produced hadron

\[ \phi_1 + \phi_2 \text{ method:} \]
hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy

\[ \phi_0 \text{ method:} \]
hadron 1 azimuthal angle with respect to hadron 2

Collins fragmentation functions

\[ R_{12}^{U/L} = \frac{N(\phi_1 + \phi_2)}{\langle N_{12} \rangle} \]
Normalised rates for Unlike-/Like-sign hadron pairs

\[ R_0^{U/L} = \frac{N(2\phi_0)}{\langle N_0 \rangle} \]
Collins FF – first result

Results based on double ratios:

**Red points:**
\[ \cos(\Phi_1 + \Phi_2) \] for moment of
Unlike-sign pion pairs over
Like-sign pion pair ratio: parameter \( A_{UL} \)

**Green points:**
\[ \cos(\Phi_1 + \Phi_2) \] for moment of
Unlike-sign pion pairs over
any Charged pion pair ratio: parameter \( A_{UC} \)

Collins fragmentation is large effect
Consistent with SIDIS indication of sign change between favoured and disfavoured Collins FF

Results used in global fit of Collins FF and transversity:
PRD **75**, 054032 (2007),
and more recently:
Kang, Prokudin, Sun and Yuan, arXiv:1505.05589

Belle results:
PRL **96**, 232002 (2006);
PRD **78**, 032011 (2008),
Err. PRD **86**, 039905 (2012)
Double ratios for K/\pi pairs – prelim.

- First pion-kaon and kaon-kaon Collins results.
- Pion-pions consistent with previous results.
- Pion-pion and kaon-kaon of similar shape and magnitude.
- Pion-kaon substantially smaller.

\[ \text{pi}^0/\text{eta-charged pion combinations will be ready to show soon as well!} \]
Asymmetries (integrated over $z$) increasing with transverse momentum.

**Belle Preliminary**

$\pi^0/\eta$-charged pion combinations will be ready to show soon as well!
Future: Belle II @ SuperKEKB

- Goal: Extremely high luminosity $\sim 10^{36} \text{ cm}^{-2}\text{s}^{-1}$ (~40x KEK) → 50-times Belle data
- Upgrades of Accelerator (Nano-beams + Higher Currents) and Detector (improved Vtx, PID, calorimetry, higher rates, modern DAQ)
- Potential for fragmentation analyses: Better PID, vertexing can help distinguish between charm and light quark events.

Commissioning of accelerator starts in 2016, data-taking in 2018
Summary and conclusions

- Unpolarised single-hadron cross sections extracted at Belle and already used in global FF fits

- First di-hadron + single proton cross sections from $e^+e^-$ extracted, soon to be submitted
  - Access to disfavoured fragmentation via ordering of pion and kaon pairs

- Transverse momentum dependent FF analysis is ongoing
  - Collins asymmetries for pions used in global transversity analysis
  - New Kaon related Collins asymmetries preliminary,
  - $\pi^0$ and eta combinations to follow soon

- Future prospects: Belle II