

Measurement of $e^+e^- \rightarrow \gamma\chi_{cJ}$ via ISR at Belle Experiment

Yanliang Han

Shandong University

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Motivation

Data Sample and Selection Rules

Backgrounds

Efficiency

$\psi(2S)$ branching fractions

High mass region study

Summary

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- The potential models predict five vector states

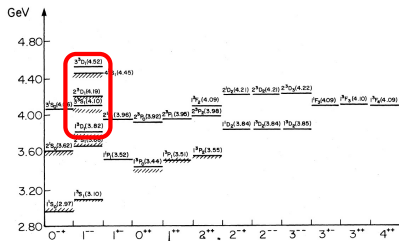


Figure: The charmonium $c\bar{c}$ [1]

[1] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985)

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- ▶ Six vector states are discovered at Experiments.

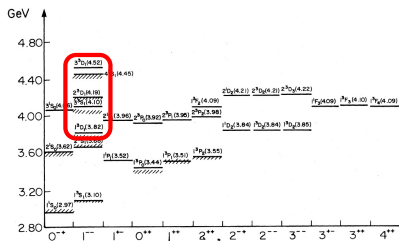
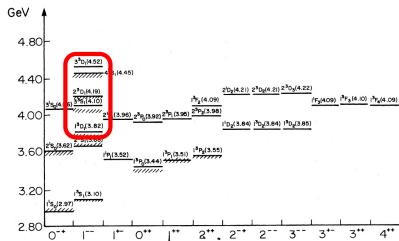


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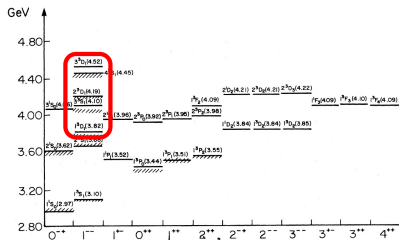


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 [2] C. Z. Yuan *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 182004 (2007)

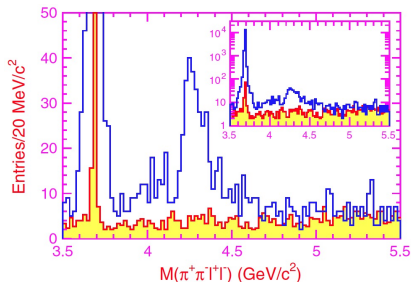


Figure: Invariant mass of $\pi^+\pi^- J/\psi$, $\Upsilon(4260)$ [2]

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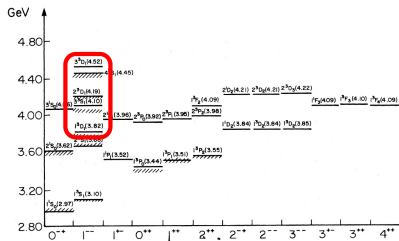


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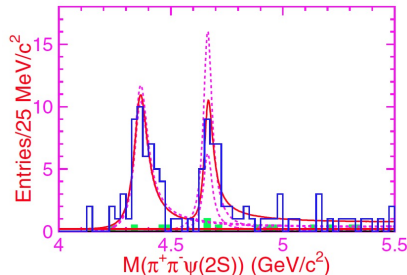


Figure: Invariant mass of $\pi^+\pi^-\psi(2S)$, $Y(4360)$ $Y(4660)$ [3]

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- ▶ Full Belle data sample
- ▶ Radiative transitions: $e^+e^- \rightarrow \gamma\chi_{cJ}$ via ISR, $\chi_{cJ} \rightarrow \gamma J/\psi$, $J/\psi \rightarrow \mu^+\mu^-$

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Data and MC Samples

DATA

- ▶ Full Belle data sample, integrated luminosity is 980 fb^{-1} .

MC Samples

- ▶ EVTGEN with the VECTORISR model is used to simulate the signal process $e^+e^- \rightarrow \gamma_{\text{ISR}} V \rightarrow \gamma_{\text{ISR}} \gamma \chi_{cJ} \rightarrow \gamma_{\text{ISR}} \gamma \gamma J/\psi$
- ▶ Background MC samples are generated with PHOKHARA

Event selection

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- ▶ Two good charged tracks with zero net charge
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- ▶ Two highest energy photons (Non ISR) in the lab. system
- ▶ Reject $\eta(\pi^0)J/\psi$ events
 - ▶ $M(\gamma\gamma)$ are not in the η mass region $[0.50, 0.58] \text{ GeV}/c^2$
 - ▶ $M(\gamma\gamma) > 0.20 \text{ GeV}/c^2$ to reject π^0 and other low invariant mass events

Missing Mass Square

Defined as $M_{\text{rec}}^2 = (P_{e^+e^-} - P_{\gamma\gamma\mu^+\mu^-})^2$

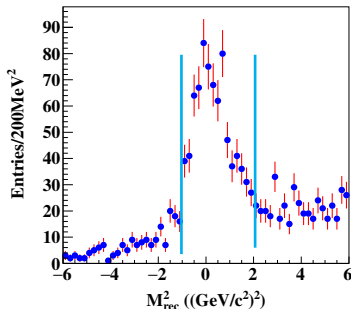


Figure: MMS distribution with $M(\gamma_I\gamma_h J/\psi) < 5.56 \text{ GeV}/c^2$

Here $M(\gamma_I\gamma_h J/\psi) = M(\gamma_I\gamma_h\mu^+\mu^-) - M(\mu^+\mu^-) + m_{J/\psi}$

We require: $-1 \text{ (GeV}/c^2)^2 < M_{\text{rec}}^2 < 2 \text{ (GeV}/c^2)^2$

Invariant mass of $\mu^+\mu^-$

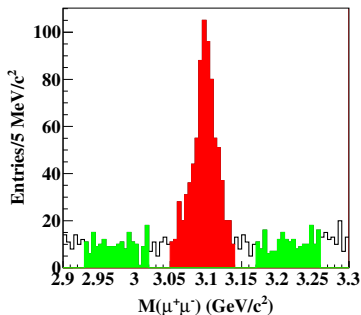


Figure: Invariant mass distribution of $\mu^+\mu^-$. The shaded area in the middle is the J/ψ signal region, and the shaded regions on both sides are the J/ψ mass sidebands.

- ▶ J/ψ signal region: Within $\pm 45 \text{ MeV}/c^2$ of the J/ψ mass
- ▶ J/ψ sideband: $[3.172, 3.262]$ or $[2.932, 3.022] \text{ GeV}/c^2$

Invariant mass of $\gamma J/\psi$

Including $M(\gamma_h J/\psi)$ and $M(\gamma_l J/\psi)$, two entries per event

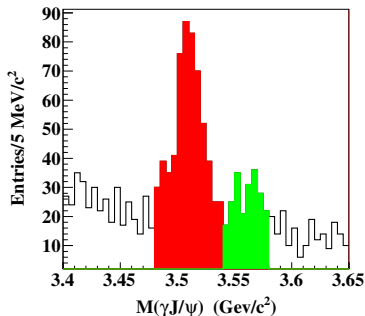


Figure: Invariant mass distribution of $\gamma J/\psi$ for candidate events with $M(\gamma_l \gamma_h J/\psi) < 5.56 \text{ GeV}/c^2$. The shaded histograms show the χ_{c1} ($[3.48, 3.535] \text{ GeV}/c^2$) and χ_{c2} ($[3.535, 3.58] \text{ GeV}/c^2$) regions.

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 - ▶ $\sigma(e^+e^- \rightarrow \pi^0 \pi^0 J/\psi) = \frac{1}{2} \sigma(e^+e^- \rightarrow \pi^+ \pi^- J/\psi)$
 - ▶ $\sigma(e^+e^- \rightarrow \pi^+ \pi^- J/\psi)$ can be got from [1]

[1] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013)

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- ▶ $e^+e^- \rightarrow \gamma_{\text{ISR}} \eta J/\psi$
 - ▶ $\sigma(e^+e^- \rightarrow \eta J/\psi)$ can be got from [2]

[1] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013)

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- ▶ $e^+e^- \rightarrow \gamma_{\text{ISR}} \eta J/\psi$
 - ▶ $\sigma(e^+e^- \rightarrow \eta J/\psi)$ can be got from [2]
- ▶ $e^+e^- \rightarrow \gamma_{\text{ISR}} \psi(2S)$ at high mass region

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Efficiency

There is only one ISR photon generated in EVTGEN. The efficiency will be over-estimated. We need to correct this.

- ▶ PHOKHARA can be used to generate $e^+e^- \rightarrow \gamma_{ISR}\eta J/\psi$ with one or two ISR photons.
- ▶ EVTGEN can also generate $e^+e^- \rightarrow \gamma_{ISR}\eta J/\psi$
- ▶ The difference is used to correct the ISR effect

The process $\psi(2S) \rightarrow \gamma \chi_{cJ} \rightarrow \gamma \gamma J/\psi$ is dominated by "E1" transition, with some mixing of "M2" and "E3". The photon angular distribution is given in [1].

This is also considered by assuming that all ISR photons is emitted from initial electrons.

[1] Karl G *et al.* Phys. Rev. D **13**, 1203 (1976)

Efficiency curve and $\gamma\gamma J/\psi$ mass

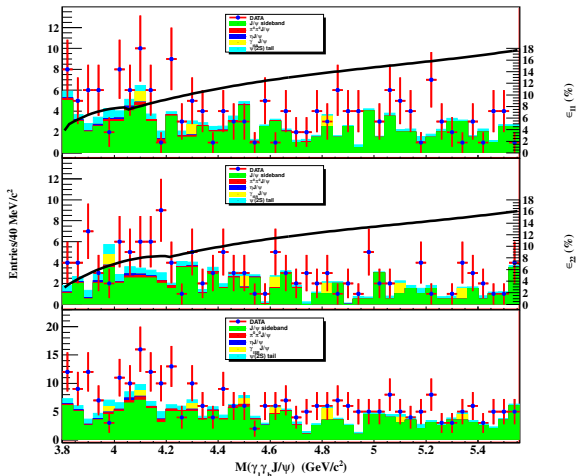


Figure: Invariant mass distributions of $\gamma\chi_{cJ}$ candidates. Shown from top to bottom are $\gamma\chi_{c1}$, $\gamma\chi_{c2}$, and their sum.

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$\psi(2S) \rightarrow \gamma\chi_{cJ}$ branching fractions

$\psi(2S)$ signal region: $3.65 < M(\gamma_I\gamma_h J/\psi) < 3.72 \text{ GeV}/c^2$.

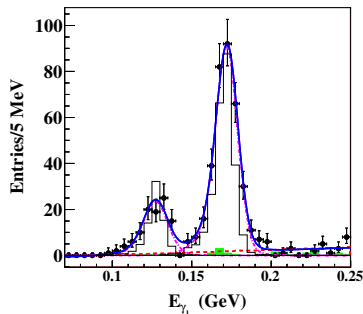


Figure: Energy distributions of the low energy photon in the $\gamma_I\gamma_h J/\psi$ CM system for events in the $\psi(2S)$ mass region.

The fit gives $340 \pm 20 \chi_{c1}$ and $97 \pm 12 \chi_{c2}$ signal events.

- ▶ $\sigma[e^+e^- \rightarrow \gamma_{\text{ISR}}\psi(2S)] = (14.25 \pm 0.26) \text{ pb}$
- ▶ $\mathcal{L} = 980 \text{ fb}^{-1}$
- ▶ $\epsilon_{\chi_{c1}} = 1.4\%$, $\epsilon_{\chi_{c2}} = 0.7\%$

$$\mathcal{B}[\psi(2S) \rightarrow \gamma\chi_{c1} \rightarrow \gamma\gamma J/\psi] = (2.92 \pm 0.19)\%$$

$$\mathcal{B}[\psi(2S) \rightarrow \gamma\chi_{c2} \rightarrow \gamma\gamma J/\psi] = (1.65 \pm 0.21)\%$$

PDG14 gives 2.93 ± 0.15 and 1.52 ± 0.15

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The high mass region: $M(\gamma\gamma J/\psi) \in [3.8, 5.56] \text{ GeV}/c^2$

There is no obvious signals, upper limits are given.

Cross section upper limit

We first construct the likelihood function $L(N^{\chi_{c1}}, N^{\chi_{c2}})$ for the number of produced events.

Using $L(N^{\chi_{c1}}, N^{\chi_{c2}})$, upper limit of $N^{\chi_{c1}}$ and $N^{\chi_{c2}}$ at 90% C.L. can be got. And then the upper limit of $\sigma(e^+e^- \rightarrow \gamma\chi_{cJ})$.

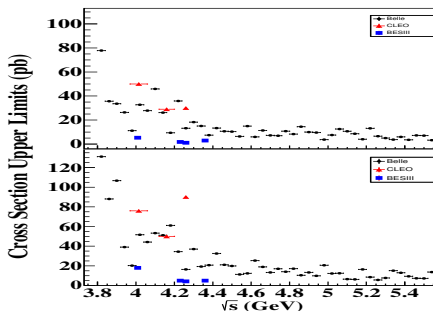


Figure: Measured upper limits on the $e^+e^- \rightarrow \gamma\chi_{cJ}$ cross sections at the 90% C.L. for χ_{c1} (top) and χ_{c2} (bottom).

Transition rate of charmonium to $\gamma\chi_{cJ}$

We can fit the mass spectrum of $\gamma\gamma J/\psi$ to get transition rate of the vector charmonium to $\gamma\chi_{cJ}$.

- ▶ One Breit-Wigner function as the signal and a linear function as the background
- ▶ The mass and total width are fixed
- ▶ $\Gamma_{ee} \times \mathcal{B}(R \rightarrow \gamma\chi_{cJ})$ is scanned to obtain the p.d.f.

Table: Upper limits on $\Gamma_{ee} \times \mathcal{B}$ at the 90% C.L.

	χ_{c1} (eV)	χ_{c2} (eV)
$\Gamma_{ee}[\psi(4040)] \times \mathcal{B}[\psi(4040) \rightarrow \gamma\chi_{cJ}]$	2.9	4.6
$\Gamma_{ee}[\psi(4160)] \times \mathcal{B}[\psi(4160) \rightarrow \gamma\chi_{cJ}]$	2.2	6.1
$\Gamma_{ee}[\psi(4415)] \times \mathcal{B}[\psi(4415) \rightarrow \gamma\chi_{cJ}]$	0.47	2.3
$\Gamma_{ee}[Y(4260)] \times \mathcal{B}[Y(4260) \rightarrow \gamma\chi_{cJ}]$	1.4	4.0
$\Gamma_{ee}[Y(4360)] \times \mathcal{B}[Y(4360) \rightarrow \gamma\chi_{cJ}]$	0.57	1.9
$\Gamma_{ee}[Y(4660)] \times \mathcal{B}[Y(4660) \rightarrow \gamma\chi_{cJ}]$	0.45	2.1

Upper limits on branching fractions

Taking $\Gamma_{ee}[\psi(4040)]$ and $\Gamma_{ee}[\psi(4415)]$ from the world average values and $\Gamma_{ee}[\psi(4160)]$ from the BES II measurement.

Table: Upper limits on branching fractions $\mathcal{B}(R \rightarrow \gamma\chi_{cJ})$ at the 90% C.L.

Resonance	$\gamma\chi_{c1} (10^{-3})$	$\gamma\chi_{c2} (10^{-3})$
$\psi(4040)$	3.4	5.5
$\psi(4160)$	6.1	16.2
$\psi(4415)$	0.83	3.9

Upper limits on branching fractions ratio

Taking $\Gamma_{ee}[Y(4260)] \times \mathcal{B}[Y(4260) \rightarrow \pi^+\pi^- J/\psi] = (6.4 \pm 0.8 \pm 0.6) \text{ eV}$
or $(20.5 \pm 1.4 \pm 2.0) \text{ eV}$ [1]

$\Gamma_{ee}[Y(4360)] \times \mathcal{B}[Y(4360) \rightarrow \pi^+\pi^-\psi(2S)] = (10.4 \pm 1.7 \pm 1.4) \text{ eV}$ or
 $(11.8 \pm 1.8 \pm 1.4) \text{ eV}$ [2]

$\Gamma_{ee}[Y(4660)] \times \mathcal{B}[Y(4660) \rightarrow \pi^+\pi^-\psi(2S)] = (3.0 \pm 0.9 \pm 0.3) \text{ eV}$ or
 $(7.6 \pm 1.8 \pm 0.8) \text{ eV}$ [2]

Table: Upper limits on branching fraction ratios at the 90% C.L.

Resonance	$\gamma\chi_{c1}$	$\gamma\chi_{c2}$
$\frac{\mathcal{B}[Y(4260) \rightarrow \gamma\chi_{cJ}]}{\mathcal{B}[Y(4260) \rightarrow \pi^+\pi^- J/\psi]}$	0.3 or 0.07	0.7 or 0.2
$\frac{\mathcal{B}[Y(4360) \rightarrow \gamma\chi_{cJ}]}{\mathcal{B}[Y(4360) \rightarrow \pi^+\pi^-\psi(2S)]}$	0.06 or 0.05	0.2 or 0.2
$\frac{\mathcal{B}[Y(4660) \rightarrow \gamma\chi_{c1}]}{\mathcal{B}[Y(4660) \rightarrow \pi^+\pi^-\psi(2S)]}$	0.2 or 0.07	0.9 or 0.3

[1] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013)

[2] X. L. Wang *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 142002 (2007)

Systematic uncertainty

- ▶ Tracking efficiency: 0.35% per track
- ▶ particle identification: 1.9%
- ▶ J/ψ and χ_{cJ} mass: 1.0% and 1.3%
 - ▶ Estimate with $\psi(2S) \rightarrow \gamma\chi_{cJ}$
- ▶ Generator
 - ▶ Corrected with $e^+e^- \rightarrow \eta J/\psi$, The difference 9.0% between measured \mathcal{B} and PDG14 is taken
 - ▶ EVTGEN: 1.0%
 - ▶ Statistical error of the MC samples: 2.0%
- ▶ Luminosity: 1.4%
- ▶ Trigger: 2%
- ▶ Branching fractions of the intermediate states: 4.5%

Assuming that all these systematic error sources are independent, the total systematic error is 13.4%.

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- ▶ The process $e^+e^- \rightarrow \gamma_{ISR}\gamma\chi_{cJ}$ is studied using full Belle data sample
- ▶ There is no obvious $\gamma\chi_{cJ}$ signal at high mass region
- ▶ The analysis is validated with branching fraction of $\mathcal{B}[\psi(2S) \rightarrow \gamma\chi_{cJ} \rightarrow \gamma\gamma J/\psi]$
- ▶ Cross section upper limit of $e^+e^- \rightarrow \gamma\chi_{cJ}$ is set at 90% C.L. between $\sqrt{s} = 3.8$ to 5.56 GeV
- ▶ Upper limits on the decay rate of the vector charmonium [$\psi(4040)$, $\psi(4160)$, and $\psi(4415)$] and charmoniumlike [$Y(4260)$, $Y(4360)$, and $Y(4660)$] states to $\gamma\chi_{cJ}$ is also set at 90% C.L.

Thank You!

Backup

Event selection

- ▶ Two good charged tracks with zero net charge
 - ▶ $P_T > 0.1 \text{ GeV}/c$
 - ▶ $|dr| < 0.5 \text{ cm}$, $|dz| < 5.0 \text{ cm}$ for charged tracks
- ▶ Muon identification is required
 - ▶ One track should satisfy $\mathcal{R}_\mu = \frac{\mathcal{L}_\mu}{\mathcal{L}_\mu + \mathcal{L}_\pi} > 0.95$
 - ▶ If $\mathcal{R}_\mu = 0$ for one track, $|\cos \theta_\mu| < 0.75$ is required for each track
- ▶ A photon candidate does not match any charged tracks
 - ▶ Photons with $E_\gamma > 3 \text{ GeV}$ in e^+e^- CM will be labeled as ISR photon (Excluded)
 - ▶ $E_\gamma > 0.25 \text{ GeV}$ in lab. system
 - ▶ Two highest energy photons in the laboratory system
- ▶ Reject $\eta(\pi^0)J/\psi$ events
 - ▶ $M(\gamma\gamma)$ are not in the η mass region $[0.50, 0.58] \text{ GeV}/c^2$
 - ▶ $M(\gamma\gamma) > 0.20 \text{ GeV}/c^2$ to reject π^0 and other low invariant mass events

Invariant mass of $\gamma_l \gamma_h J/\psi$

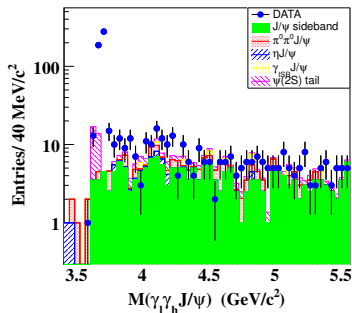


Figure: Invariant mass distribution of $\gamma_l \gamma_h J/\psi$. The background from the tail of the $\psi(2S)$ is plotted only for $M(\gamma_l \gamma_h J/\psi) > 3.75 \text{ GeV}/c^2$ and $M(\gamma_l \gamma_h J/\psi) < 3.65 \text{ GeV}/c^2$.

Cross section upper limit

Maximum likelihood is used to get the upper limits.

The numbers of the expected signal events, $\nu^{\chi_{c1}}$ and $\nu^{\chi_{c2}}$

$$\begin{pmatrix} \nu^{\chi_{c1}} \\ \nu^{\chi_{c2}} \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{21} \\ \epsilon_{12} & \epsilon_{22} \end{pmatrix} \begin{pmatrix} N^{\chi_{c1}} \times \mathcal{B}(\chi_{c1} \rightarrow \gamma J/\psi) \times \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-) \\ N^{\chi_{c2}} \times \mathcal{B}(\chi_{c2} \rightarrow \gamma J/\psi) \times \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-) \end{pmatrix}$$

The numbers of expected events

$$\begin{pmatrix} \mu^{\chi_{c1}} \\ \mu^{\chi_{c2}} \end{pmatrix} = \begin{pmatrix} \nu^{\chi_{c1}} \\ \nu^{\chi_{c2}} \end{pmatrix} + \begin{pmatrix} n_{\text{bkg}}^{\chi_{c1}} \\ n_{\text{bkg}}^{\chi_{c2}} \end{pmatrix},$$

The probability of observing $(n_{\text{obs}}^{\chi_{c1}}, n_{\text{obs}}^{\chi_{c2}})$

$$p(N^{\chi_{c1}}, N^{\chi_{c2}}) = \frac{(\mu^{\chi_{c1}})^{n_{\text{obs}}^{\chi_{c1}}} e^{-\mu^{\chi_{c1}}}}{n_{\text{obs}}^{\chi_{c1}}!} \frac{(\mu^{\chi_{c2}})^{n_{\text{obs}}^{\chi_{c2}}} e^{-\mu^{\chi_{c2}}}}{n_{\text{obs}}^{\chi_{c2}}!}$$

Considering the uncertainty in the background estimation and systematic error

$$L(N^{\chi_{c1}}, N^{\chi_{c2}}) = \sum_{k,l,m,n} p(N^{\chi_{c1}}, N^{\chi_{c2}}) = \sum_{k,l,m,n} \frac{(\mu_{k,l}^{\chi_{c1}})^{n_{\text{obs}}^{\chi_{c1}}} e^{-\mu_{k,l}^{\chi_{c1}}}}{n_{\text{obs}}^{\chi_{c1}}!} \frac{(\mu_{m,n}^{\chi_{c2}})^{n_{\text{obs}}^{\chi_{c2}}} e^{-\mu_{m,n}^{\chi_{c2}}}}{n_{\text{obs}}^{\chi_{c2}}!}$$