Fragmentation Functions at Belle

Martin Leitgab (University of Illinois) for the Belle collaboration
1. Motivation & Experimental Methods

- **Fragmentation Functions (FF)** input quantities for exploration of nucleon spin structure in SIDIS & pp
- **Extract FF via pQCD** analysis from SIDIS, pp and e+e- measurements
- For **high precision** measurements: use large dataset of **Belle experiment** at KEK-B, Japan:
  - KEK-B: asymmetric e+ (3.5 GeV) e- (8 GeV) collider
  - world record inst. luminosity > 2.1 e+34 cm-2s-1
  - $\sqrt{s} = 10.58 \text{ GeV} (\Upsilon(4S)) \, e^+e^- \to \Upsilon(4S) \to B \bar{B}$
  - Continuum production 10.52 GeV, $e^+e^-\to q \bar{q} \ (u,d,s,c)$
  - Belle collected luminosity: $\sim 1000 \text{ fb}^{-1}$; $\sim 90 \text{ fb}^{-1}$ continuum
  - Tracking acceptance $\theta \ [17^\circ;150^\circ]$
  - PID: Drift Chamber, Cherenkov, ToF, EMcal, Mu detector
2. Extract parton helicity distributions with unpolarized Fragmentation Functions

- Unpolarized FF input quantities for extraction of quark & antiquark helicity distributions in SIDIS (Hermes, JLab 12 GeV) and
- extraction of gluon helicity distribution in pp (RHIC experiments):

\[ p^\uparrow + p^\downarrow \rightarrow \pi^+ + X \]

\[ d^3 \Delta \sigma(p^\uparrow p^\uparrow \rightarrow \pi^+ X) \frac{dx_1 dx_2 dz}{dx_1 dx_2 dz} \propto \Delta G(x_1) \Delta q(x_2) \times d^2 \sigma(gq \rightarrow gq) \frac{dx_1 dx_2}{dx_1 dx_2} \times D_q^{\pi^+}(z) + \ldots \]

measured at RHIC
2. Unpolarized Fragmentation Functions-Current Extractions


- Quark FF well constrained, limited constraint on gluon FF- can be improved by precision measurement at low $Q^2$ in addition to large available datasets from LEP
2. Unpolarized Fragmentation Functions - New precision measurement at $Q^2 = 100 \text{ GeV}^2$ in progress at Belle

- Largest uncertainty expected from **PID effects**:
  - Completed **data-based PID correction** covering most of kinematic range

\[ N_j = P \cdot N_i \]

PID correction by inversion of PID probability matrix.

\[
\begin{array}{cccc}
p(e \rightarrow \pi) & p(\pi \rightarrow e) & p(\pi \rightarrow \mu) & p(\pi \rightarrow K) \\
p(\mu \rightarrow \pi) & p(\mu \rightarrow \mu) & p(\mu \rightarrow K) & p(\mu \rightarrow p) \\
p(\pi \rightarrow e) & p(\pi \rightarrow \mu) & p(\pi \rightarrow K) & p(\pi \rightarrow p) \\
p(\mu \rightarrow e) & p(\mu \rightarrow \mu) & p(\mu \rightarrow K) & p(\mu \rightarrow p) \\
\end{array}
\]

ToF forward geometry acceptance limit
\((-0.5 < \cos \theta_{\text{lab}} < 0.8)\)

scatter plot: $e, \mu, \pi, K$ and $p$ tracks from $4 \times 10^5$ events
2. Unpolarized Fragmentation Functions—New precision measurement at $Q^2 = 100 \text{ GeV}^2$ in progress at Belle

Extensive PID studies PID probabilities extracted from real data over wide kinematic range:

e.g. $D^* \rightarrow \pi_{\text{slow}} (D^0 \rightarrow \pi_{\text{fast}} K)$

Further corrections for momentum smearing, acceptance effects
2. Unpolarized Fragmentation Functions-Projected Belle sensitivities

- Measurement will give **precision data** set for $Q^2 = 100 \text{ GeV}^2$ (esp. at high $z$):

![Projected relative statistical and systematic uncertainties](image)

$e^+e^- \rightarrow h^+ X$ data
3. Chiral-odd Fragmentation Functions for the extraction of transversity distribution

- To determine transverse quark spin distributions:
  \[ \text{pp, SIDIS measure:} \]
  \[ \text{transversity} \times \text{Collins FF:} \quad \delta q \times H_1^q \]
  or
  \[ \text{transversity} \times \text{Interference FF (IFF):} \quad \delta q \times H_1^{q} \]

- **Belle** can measure: Collins \times Collins or IFF \times IFF by identifying two/four hadrons in **two hemispheres** in final state, \( e^+e^- \rightarrow (h)(h)X \) / \( (hh)(hh)X \)

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Event topology in
\[ e^+e^- \rightarrow q\bar{q} \rightarrow (h)(h)X \]
3a. Collins Fragmentation Functions $H_1^-$ at Belle

  
  fragmentation of polarized quark into unpolarized hadron:
  azimuthal dependence of hadron production cross-section
  \[ \propto (\vec{k} \times \vec{p}_{h\perp}) \cdot \vec{s}_q \]
  \[ \propto \sin \phi \]

- In unpolarized $e^+e^-$ quark spin unknown -> Collins effect would average out on single jet -> measure simultaneously in both hemispheres (quark spins correlated)

2-hadron inclusive cross section:
\[
\frac{d\sigma(e^+e^- \to h_1h_2X)}{d\Omega dz_1dz_2} \propto \sin^2 \Theta \cos(\varphi_1 + \varphi_2) H_{1}^{\perp[1]}(z_1) \overline{H}_{1}^{\perp[1]}(z_2)
\]
3a. Collins Fragmentation Functions at Belle

- Measure ‘double ratios’ of normalized two-pion yields -> access to favored-disfavored Collins FF (transverse spin asymmetries)


- First direct measurement of Collins effect

- Significant, nonzero asymmetries implying significant spin-dependent effects in $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s$) fragmentation

- This measurement used in first extraction of quark transversity distributions:
  Phys. Rev. D 75, 054032 (2007),
3b. Interference Fragmentation Functions

- **Interference FF**: Collins, Heppelmann, Ladinsky NPB420 (1994)
  - fragmentation of polarized quark into two hadrons correlated via partial wave interference;
  - again azimuthal dependence of hadron production $\propto (\vec{k} \times \vec{R}) \cdot \vec{s}_q$

- **Benefits of IFF in transversity extraction**: alternative approach to Collins FF extraction
  - (most likely) spin correlation persists even after integrating over transverse momenta of hadronic final state: collinear schemes usable for factorization & pQCD evolution (vs. Collins FF)
3b. Interference Fragmentation Functions at Belle

• Again asymmetry would average out on single jet (unpolarized initial state at Belle, quark spin state not known)

• -> measure simultaneous azimuthal modulation of two pion pair yields in opposing hemispheres

\[
A \propto \cos(\phi_1 + \phi_2) H_1^\perp(z_1, m_1) \overline{H}_1^\perp(z_2, m_2)
\]

Model predictions for IFF from:

Jaffe et al PRL 80, (1998)
Bacchetta et al. PRD 79, (2009)
3b. Interference Fragmentation Functions at Belle – Measurement results

Measurement results 8x8 $m_1 \ m_2$ binning
3b. Interference Fragmentation Functions at Belle- Model predictions

Blue points: data (after sign change and cos moment, decay angular moments adjustment)

- Model prediction: IFF larger around invariant mass of correlating resonance, then fall-off
- Measured: ~ constant IFF at high invariant masses, raising IFF for increasing fractional energy
4. FFs at Belle- Summary & Outlook

• Measurement in progress of high precision datasets for extraction of unpolarized FF, input for helicity structure measurements; preliminary results expected end of 2010.

• First direct measurement of Collins FF, large significant asymmetries found, data already used in transversity extraction on SIDIS data

• First direct measurement of Interference FF, also significant asymmetries rising with invariant mass and fractional energy, for complementary extraction of quark transversity distributions

• Future high precision measurements of FFs at Belle:
  - $k_t$ dependence of Collins FF
  - chiral-odd $\Lambda$ FF, $\Lambda$ single spin asymmetries
  - $k_t$ dependence of unpolarized FF
  - unpolarized di-hadron FF
Backup
2. Unpolarized Fragmentation Functions

- FF $D_i^h$ describes probability for a parton $i$ to fragment into a hadron $h$:
  
  \[ F^h(z, Q^2) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} \]

- FFs $D_i^h$ can be extracted from e+e- data via:
  
  \[ z = \frac{E_{h}^{\text{cms}}}{\sqrt{s}/2} \]

measured: hadron multiplicity

\[
\text{LO QCD} = \sum_{i=q,\bar{q}} Q_q \cdot D_i^h(z, Q^2)
\]

\[
\text{NLO QCD} = \sum_{i=q,\bar{q},g} C_i^{\text{NLO}}(z, \alpha_s) \otimes D_i^h(z, Q^2)
\]

extracted: Parton FFs

\[
= \sum_{i=q,\bar{q},g} \int_{z}^{1} \frac{dz'}{z'} C_i(z', \alpha_s(\mu), \frac{s}{\mu^2}) D_i^h \left( \frac{z}{z'}, \mu^2 \right) + O\left( \frac{1}{\sqrt{s}} \right)
\]
**FF Extractions by HKNS & DSS**

- **Method of extraction:**
  - parameterization of $D_i^h$ at initial scale $Q_0^2$

  **HKNS**
  $$D_i^h(z, Q_0^2) = \frac{1}{B[2 + \alpha_i^h, \beta_i^h + 1]} M_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h}$$

  **DSS**
  $$D_i^h(z, Q_0^2) = \frac{1}{B[2 + \alpha_i^h, \beta_i^h + 1] + \gamma_i^h B[2 + \alpha_i^h, \beta_i^h + \delta_i^h + 1]} M_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h} [1 + \gamma_i^h (1-z)^{\delta_i^h}]$$

- evolution of (scale-dependent) $D_i^h$ to $Q^2$ of fitted datasets via pQCD
  DGLAP evolution equations and simulataneous fit of all datasets.

  $$\frac{d}{d \ln Q^2} \bar{D}^h(z, Q^2) = [\hat{P}^{(T)} \otimes \bar{D}^h](z, Q^2);$$

  $$\bar{D}^h \equiv \begin{pmatrix} D_{\Sigma}^h \\ D_q^h \\ D_{\bar{q}}^h \\ D_g^h \end{pmatrix}, \quad D_{\Sigma}^h \equiv \sum_q (D_q^h + D_{\bar{q}}^h), \quad \hat{P}^{(T)} \equiv \begin{pmatrix} 1 \\ P_{qq}^{(T)} \end{pmatrix} 2n_f P_{gq}^{(T)} \begin{pmatrix} P_{gg}^{(T)} \end{pmatrix}.$$
Fit Results HKNS

TABLE VII: Each $\chi^2$ contribution in the pion analysis.

<table>
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<tr>
<th>experiment</th>
<th># of data</th>
<th>$\chi^2$ (LO)</th>
<th>$\chi^2$ (NLO)</th>
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<td>52.1</td>
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<td>TOPAZ</td>
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<td>2.6</td>
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<td>SLD (all)</td>
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<td>DELPHI (all)</td>
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<td>(/d.o.f.)</td>
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<td>(1.81)</td>
<td>(1.75)</td>
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Fit Results DSS

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<td>$\pi^-$, $\langle \eta \rangle = 2.95$</td>
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**TOTAL:**

<p>| | | | |</p>
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<td></td>
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<td>392</td>
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</table>
Loose constraints at low $Q^2$ - high uncertainties, esp. Gluon FF - remedied by precision measurement at low $Q^2$

Most of fitted data at $Q^2 = M_z^2$ - lower uncertainties
Extracted FF DSS

Inclusion of pp and SIDIS data provide better constraints at low $Q^2$ & high $z$. Gluon FF still loosely constrained because of large theoretical uncertainties in pp- remedied by precision measurement at low $Q^2$. 

\[ zD_1^{\pi^+}(z) \quad Q^2 = 10 \text{ GeV}^2 \]

\[ zD_1^{\pi^+}(z) \quad Q^2 = 10 \text{ GeV}^2 \]
Suggested Improvement of Constraints (esp Gluon FF) by Precision Measurement at low $Q^2$

HKNS experimental data

World $e^+e^- \rightarrow h^+X$ data

high precision hadron measurement at low $Q^2$
3. Systematic Studies- Particle Misidentification Correction

- linear model: performance of Belle PID described by
  - particle identification probabilities $p_{(i \rightarrow j)}$:
    probability that particle of species $i$ PID-selected as particle of species $j$.
  - Goal: extract $p_{(i \rightarrow j)}$ from real data.

\[ \tilde{N}_j = \hat{P} \tilde{N}_i \]

\[ [P]_{ij} = \begin{pmatrix}
  p(e \rightarrow \bar{e}) & p(\mu \rightarrow \bar{e}) & p(\pi \rightarrow \bar{e}) & p(K \rightarrow \bar{e}) & p(p \rightarrow \bar{e}) \\
  p(e \rightarrow \bar{\mu}) & p(\mu \rightarrow \bar{\mu}) & p(\pi \rightarrow \bar{\mu}) & p(K \rightarrow \bar{\mu}) & p(p \rightarrow \bar{\mu}) \\
  p(e \rightarrow \bar{\pi}) & p(\mu \rightarrow \bar{\pi}) & p(\pi \rightarrow \bar{\pi}) & p(K \rightarrow \bar{\pi}) & p(p \rightarrow \bar{\pi}) \\
  p(e \rightarrow \bar{K}) & p(\mu \rightarrow \bar{K}) & p(\pi \rightarrow \bar{K}) & p(K \rightarrow \bar{K}) & p(p \rightarrow \bar{K}) \\
  p(e \rightarrow \bar{p}) & p(\mu \rightarrow \bar{p}) & p(\pi \rightarrow \bar{p}) & p(K \rightarrow \bar{p}) & p(p \rightarrow \bar{p})
\end{pmatrix} \]

\[ N_i = P^{-1} \tilde{N}_j : \text{correction through inversion of matrix.} \]
3. Particle Misidentification Correction—PID Probability Extraction Method

- extraction of PID probabilities $p_{(i\rightarrow j)}$ ('calibrating' the PID) from real data.

- Procedure (adopted from real data PID studies, e.g. BN 321, 779, 670)
  a) create clean sample of "$i$" - particles by exploiting decay kinematics,
  b) remove kombinatoric background by fitting a signal peak,
  c) calculate probabilities: apply cuts on track PID likelihoods and build ratios.

\[ p_{(K^- \rightarrow \pi^-)} \approx 0.111 \pm 0.004 \]

\[ m_{D^*} - m_{D^0} \text{ for kinematically-selected } K^- \text{ tracks with } p_{\text{lab}} \text{ in } [1.4; 1.6] \text{ GeV/c, } \cos \theta_{\text{lab}} \text{ in } [0.02; 0.21] \text{ with } \pi \text{ PID cut} \]

\[ m_{D^*} - m_{D^0} \text{ for kinematically-selected } K^- \text{ tracks with } p_{\text{lab}} \text{ in } [1.4; 1.6] \text{ GeV/c, } \cos \theta_{\text{lab}} \text{ in } [0.02; 0.21] \text{ without PID cut} \]
3. Particle Misidentification Correction–
Lepton PID Probabilities from $J/\Psi \rightarrow \ell^+ \ell^-$

- $e^-/\mu^-$

**Example fits**

- $e^-/\mu^-$ with $p_{\text{lab}}$ in [1.4; 1.6] GeV/c, $\cos \theta_{\text{lab}}$ in [0.21; 0.36]

- $e$ and $\mu$ components not resolvable with kinematics only. S/N too low for KPID and pPID.

**Cumulative Chisquare**

- Distribution of all chisquares fitted with theoretical Chisquare distribution.

Real data exp[7;55], HadronBJ onres ~590 fb$^{-1}$, cont ~70fb$^{-1}$
3. Particle Misidentification Correction—Lepton PID Probabilities from $J/\Psi \rightarrow \ell^+\ell^-$

- Lepton PID probability samples.
- vs $p_{\text{lab}}$ and $\cos\theta_{\text{lab}}$.

Real data exp[7;55], onres $\sim$590 fb$^{-1}$, cont $\sim$70fb$^{-1}$
3. Particle Misidentification Correction–Pion and Kaon PID Probabilities from D*

\[ D^* \rightarrow \pi_\text{slow} \left( D^0 \rightarrow \pi_\text{fast} K \right) \]

**Example fits**

\[ \pi^- \]

- \( p_{\text{lab}} \) in [1.4; 1.6] GeV/c, \( \cos \theta_{\text{lab}} \) in [0.21; 0.36]

Candidate selection along BN 779, Nishida-san. Good S/N.

\[ K^- \]

- \( p_{\text{lab}} \) in [1.4; 1.6] GeV/c, \( \cos \theta_{\text{lab}} \) in [0.21; 0.36]

Real data exp[7;55], HadronBJ onres ~590 fb^-1, cont ~70fb^-1
3. Particle Misidentification Correction–
Pion and Kaon PID Probabilities from D* 

Cumulative Chisquare

\[ \pi^- \]

distribution of all chisquares fitted with theoretical Chisquare distribution.

\[ K^- \]

Real data exp[7;55], HadronBJ onres ~590 fb^{-1}, cont ~70 fb^{-1}
3. Particle Misidentification Correction—
Pion and Kaon PID Probabilities from D*

- Pion PID probability samples.
- Good kinematic availability.

Real data exp[7;55],
onres ~580 fb^{-1}, cont ~65 fb^{-1}
3. Particle Misidentification Correction—Pion and Kaon PID Probabilities from D*

- Kaon PID probability samples.
- Good kinematic availability.

Real data exp[7;55], onres ~580 fb^-1, cont ~65fb^-1
3. Particle Misidentification Correction—Proton PID Probabilities from Λ → p π

Example fits

p- with
p_{lab} in [1.4; 1.6] GeV/c,
\cos{\theta}_{lab} in [0.21; 0.36]

candidate selection along
BN 670 (Majumder-san),
BN 1126 (Yeh-san).

Good S/N, but candidate selection Ks- cuts
-> non-linear background.

Cumulative Chisquare
distribution of all chisquares fitted with theoretical Chisquare distribution.

Real data exp[7;55], HadronBJ
MDst_vey2; onres ~590 fb^-1, cont ~70fb^-1
3. Particle Misidentification Correction—
Proton PID Probabilities from $\Lambda \rightarrow p \pi$

- Proton PID probability samples.
- vs $p_{\text{lab}}$ and $\cos \theta_{\text{lab}}$.

Real data exp[7;55], HadronBJ Mdst_vee2; onres $\sim 590$ fb$^{-1}$, cont $\sim 70$ fb$^{-1}$
Backup: Particle Misidentification Correction—Lepton PID Probabilities from \( J/\Psi \to \ell^+\ell^- \)

Real data exp[7;55], onres ~590 fb^{-1}, cont ~70 fb^{-1}

**Kinematic extraction availability**

- **Red/blue:** good fits.
- **Green:** failed fits (no signal)
- **White:** by-hand-rejected.

Restricted kinematic phase space for decay leptons (J/\Psi from B).

**Consistency Check**

Agreement between the two methods within 1 sigma for all extracted probabilities.
Backup: Particle Misidentification Correction—Pion and Kaon PID Probabilities from D*

Kinematic extraction availability

π⁻

- Red/blue: good fits.
- Green: failed fits (no signal)
- White: by-hand-rejected.

Good availability.

K⁻

Real data exp[7;55], HadronBJ onres ~590 fb⁻¹, cont ~70 fb⁻¹

K PID cut

p PID cut

Unsel.
Backup: Particle Misidentification Correction—Pion and Kaon PID Probabilities from D*

Consistency check

**π**⁻

Agreement between the two methods within 1 sigma for all extracted probabilities.

**K**⁻

Real data exp[7;55], HadronBJ onres ~590 fb⁻¹, cont ~70 fb⁻¹
Backup: Particle Misidentification Correction—Proton PID Probabilities from $\Lambda \rightarrow p\pi$

Kinematic extraction availability

- Red/blue: good fits.
- Green: failed fits (no signal)
- White: by-hand-rejected.

Good availability.

Consistency Check

- $\sigma(\delta)$
Backup:
Particle Misidentification Correction–Extrapolation of Real Data PID Probabilities

- Interpolate small gaps with real data PID probability
- Extrapolate high $p_{lab}$ probabilities in high $p_{lab}$ bins with Monte Carlo help.
- Color points: real data probabilities, gray points: generic MC probabilities.
Backup. PID Probability Extraction

- Probability extraction $p(i \rightarrow j)$: building ratios between signal yields - integrated signal functions.

\[ p_{i\rightarrow j} = \frac{N_{i\rightarrow j}}{N_i} = \frac{\int_{m_{\text{low}}}^{m_{\text{high}}} g_{D_i \rightarrow j(m)} (\overline{D}_i \rightarrow j(m), m) \, dm}{\text{histbinwidth}} \]

\[ = \frac{\int_{m_{\text{low}}}^{m_{\text{high}}} f_{D_i(m)} (\overline{D}_i(m), m) \, dm}{\text{histbinwidth}} \]
Backup. PID Probability Extraction

- Probability uncertainty: binomial interpretation for Ni->j

\[
\begin{align*}
(1) \quad p_{i \rightarrow j} &= \frac{N_{i \rightarrow j}}{\int_{m_{\text{low}}}^{m_{\text{high}}} f_{\text{sig}}(\text{par}_{\bar{D}(m)}, m) \, dm}. \\
(2) \quad \sigma_{N_{i \rightarrow j}}^2 &= p_{i \rightarrow j}(1 - p_{i \rightarrow j})N_i. \\
(3) \quad \sigma_{p_{i \rightarrow j}, N_{i \rightarrow j}}^2 &= \left(\frac{1}{\int_{m_{\text{low}}}^{m_{\text{high}}} f_{\text{sig}}(\text{par}_{\bar{D}(m)}, m) \, dm} \right)^2 p_{i \rightarrow j}(1 - p_{i \rightarrow j})N_i. \\
(4) \quad \sigma_{p_{i \rightarrow j}, N_i}^2 &= \sum_{k=1}^{n_{\text{sig}}} \left[ \frac{\partial}{\partial \text{par}_k} \bar{D}(m) \right]^2 \left( \frac{N_{i \rightarrow j}}{\int_{m_{\text{low}}}^{m_{\text{high}}} f_{\text{sig}}(\text{par}_{\bar{D}(m)}, m) \, dm} \right)^2 \sigma_{\text{par}_k}^2.
\end{align*}
\]
In initial test study, following BN 390,

* QED background was neglected;
  * Beam-gas background was also neglected, since only tracks within the ToF geometry acceptance and with $z < 0.75$ were considered in our analysis.

upper bound is estimated for tautau (gamma gamma) events: **additional directed systematic uncertainty of -1.8% (-1.4%) per z-bin.**
Backup - Binning for Particle Misidentification Correction

$p_{lab}$ binning:

- bin 0 = 0.5 - 0.65 GeV
- bin 1 = 0.65 - 0.8 GeV
- ... widths of 200 MeV
- bin 12 = 2.8 – 3.0 GeV
- bin 13 = 3.0 – 3.5 GeV
- bin 14 = 3.5 – 4.0 GeV
- bin 15 = 4.0 – 5.0 GeV
- bin 16 = 5.0 – 8.0 GeV

$\cos\theta_{lab}$ binning:

- bin 0: -0.511 – -0.300
- bin 1: -0.300 – -0.152
- bin 2: -0.152 – 0.017
- bin 3: 0.017 – 0.209
- bin 4: 0.209 – 0.355
- bin 5: 0.355 – 0.435
- bin 6: 0.435 – 0.542
- bin 7: 0.542 – 0.692
- bin 8: 0.692 – 0.842

$\theta_{lab}$ binning within ToF acceptance

- ToF $\theta_{lab}$ [34;120)$^\circ$
- $\cos\theta_{lab}$ in [0.829;-0.5).

$\theta_{lab}$ binning reflects ACC hardware segmentation.
# Backup – Likelihood Cuts for Particle Selection

<table>
<thead>
<tr>
<th>Selected Species $j$</th>
<th>Cuts on Likelihood Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_{\text{e} \text{i} \text{d}}$</td>
</tr>
<tr>
<td>e</td>
<td>[0.85; 1.0]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>[0.0; 0.85]</td>
</tr>
<tr>
<td>$\pi$</td>
<td>[0.0; 0.85]</td>
</tr>
<tr>
<td>K</td>
<td>[0.0; 0.85]</td>
</tr>
<tr>
<td>p</td>
<td>[0.0; 0.85]</td>
</tr>
</tbody>
</table>

![Diagram](image)
Backup: CDC, ToF and ACC PID discrimination power

- CDC discrimination power via $dE/dx$

- ToF discrimination power via time-of-flight

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Track Momentum [GeV/c]} & \text{Times-of-Flight for Particle Species } i \ [\text{ns}] \\
\hline
& e & \mu & \pi & K & p \\
1.0 & 4.004 & 4.209 & 4.274 & 4.892 & 5.572 \\
2.0 & 4.003 & 4.008 & 4.013 & 4.123 & 4.421 \\
3.0 & 4.003 & 4.005 & 4.007 & 4.057 & 4.194 \\
\hline
\end{array}
\]

- ACC discrimination power via Cerenkov thresholds

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Refractive Index} & \text{Cherenkov Threshold Momentum for Particle Species } i \\
& e \ [\text{MeV/c}] & \mu \ [\text{GeV/c}] & \pi \ [\text{GeV/c}] & K \ [\text{GeV/c}] & p \ [\text{GeV/c}] \\
1.010 & 3.6 & 0.75 & 0.98 & 3.48 & 6.62 \\
1.013 & 3.2 & 0.65 & 0.86 & 3.05 & 5.80 \\
1.015 & 2.9 & 0.61 & 0.80 & 2.84 & 5.40 \\
1.020 & 2.54 & 0.53 & 0.86 & 2.46 & 4.67 \\
1.028 & 2.14 & 0.44 & 0.59 & 2.07 & 3.94 \\
\hline
\end{array}
\]
Backup - Accounting for Probability Nature of Matrix Elements in Uncertainty Propagation through Matrix Inversion for Sample PID Matrix

- green: all MC variants of original PID matrix.
- black: rejecting all variant matrices which are not complying with probability interpretation (columnsum > 1.0 || matrix element < 0.0).