Time-dependent CPV and mixing at B-factories

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2012 Jan. 9th
KM unitarity triangle and CPV parameter convention

\[ V = \begin{pmatrix} \nu_{ud} & \nu_{us} & \nu_{ub} \\ \nu_{cd} & \nu_{cs} & \nu_{cb} \\ \nu_{td} & \nu_{ts} & \nu_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \lambda^2/2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix} \]

by Wolfenstein parametrization

Irreducible complex phase causes CP Violation (CPV)!

Comprehensive test; measure all the angles and sides.

B system: very good place, all the angles are \( O(0.1) \)!

\[ b \rightarrow u V_{ud} V_{ub}^* \text{ transition} \]

\[ B^0 - \overline{B}^0 \text{ mixing} \]

\[ V_{td} V_{tb}^* \]
Angle measurements and mixing

Decay via $b \to c$ (tree)

How about $b \to s$ (penguin)??

$b \to d$ (penguin) is also participating in some cases → direct CPV.
Time-dependent CPV

In order to see CPV by interference between decay and mixing.

\[ \Delta z = \beta \gamma c \Delta t, \]
\[ \beta \gamma = 0.425 \text{(KEKB)}, 0.56 \text{(PEP-II)} \]

\[ \Delta z \sim 200 \mu m \]

\[ A_{CP}(\Delta t) = \frac{\frac{\Gamma(\bar{B}^0(\Delta t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(\Delta t) \rightarrow f_{CP}) + \Gamma(\bar{B}^0(\Delta t) \rightarrow f_{CP})}}{\bar{A}(f_{CP})} = S_{f_{CP}} \sin(\Delta m \Delta t) + A_{f_{CP}} \cos(\Delta m \Delta t) \]

\[ S_{f_{CP}} = \frac{2 \quad \text{Im}(\lambda)}{||\lambda||^2 + 1} \quad A_{f_{CP}} = \frac{||\lambda||^2 - 1}{||\lambda||^2 + 1} \quad \lambda = \frac{q}{p} \frac{\bar{A}(f_{CP})}{A(f_{CP})} \]

\[ ||\lambda|| = 1 \quad \text{if no DCPV} \]
In order to perform such studies

B meson is so heavy that many decay modes are available. Branching fraction to the modes usable for CPV is limited. → Huge ($O(10^8)$) amount of B mesons is necessary. → Measurement of time evolution of B meson pair is required.
Two B-factories at KEK and SLAC

8 GeV(e⁻) × 3.5 GeV(e⁺),
\[ L_{\text{max}} = 2.1 \times 10^{34} \text{cm}^{-2} \text{s}^{-1} \]

9 GeV(e⁻) × 3.1 GeV(e⁺),
\[ L_{\text{max}} = 1.2 \times 10^{34} \text{cm}^{-2} \text{s}^{-1} \]
Integrated luminosity of B factories

\begin{itemize}
  \item \textbf{KEKB} \hfill \textbf{PEP-II}
  \end{itemize}

\begin{itemize}
  \item \textbf{772M BB} \quad > 1 \text{ ab}^{-1}
    \begin{itemize}
      \item On resonance: \begin{itemize}
          \item $\Upsilon(5S)$: 121 fb$^{-1}$
          \item $\Upsilon(4S)$: 711 fb$^{-1}$
          \item $\Upsilon(3S)$: 3 fb$^{-1}$
          \item $\Upsilon(2S)$: 25 fb$^{-1}$
          \item $\Upsilon(1S)$: 6 fb$^{-1}$
        \end{itemize}
      \item Off resonance: \begin{itemize}
          \item \sim 100 fb$^{-1}$
        \end{itemize}
    \end{itemize}

  \item \textbf{470M BB} \quad \sim 550 \text{ fb}^{-1}
    \begin{itemize}
      \item On resonance: \begin{itemize}
          \item $\Upsilon(4S)$: 433 fb$^{-1}$
          \item $\Upsilon(3S)$: 30 fb$^{-1}$
          \item $\Upsilon(2S)$: 14 fb$^{-1}$
        \end{itemize}
      \item Off resonance: \begin{itemize}
          \item \sim 54 fb$^{-1}$
        \end{itemize}
    \end{itemize}
\end{itemize}
sin2φ₁ at Belle
(772M BB, final sample)

Signal yield increased more than $N_{BB}$ compared to the previous publication (PRL98,031802), thanks to the data reprocessing with improved tracking.

<table>
<thead>
<tr>
<th></th>
<th>$J/ψ K_S$</th>
<th>$J/ψ K_L$</th>
<th>$ψ (2S) K_S$</th>
<th>$χ_{c1} K_S$</th>
<th>$N_{BB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sig}$</td>
<td>12727±115</td>
<td>10087±154</td>
<td>1981±46</td>
<td>943±33</td>
<td>772 M</td>
</tr>
<tr>
<td>Purity(%)</td>
<td>97</td>
<td>63</td>
<td>93</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td>$N_{sig}$(prev.)</td>
<td>7484±87</td>
<td>6512±123</td>
<td>N/A</td>
<td>N/A</td>
<td>535 M</td>
</tr>
<tr>
<td>Purity(%) (prev.)</td>
<td>97</td>
<td>59</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
sin2ϕ₁ at Belle (772M BB)

B decay mode | S_f  | A_f
---|---|---
J/ψK_S^0    | 0.671 ± 0.029 | -0.014 ± 0.021
ψ(2S)K_S^0  | 0.739 ± 0.079 | 0.103 ± 0.055
χc₁K_S^0    | 0.636 ± 0.117 | -0.023 ± 0.083
J/ψK_L^0    | -0.641 ± 0.047 | 0.019 ± 0.026

sin2ϕ₁ = 0.668 ± 0.023 ± 0.013
A_fCP = 0.007 ± 0.016 ± 0.013
\( \sin 2\phi_1 (= \sin 2\beta) \) at BaBar (465M BB)

- \( B^0 \rightarrow J/\psi K^0 \)
- \( B^0 \rightarrow \psi(2S)K^0 \)
- \( B^0 \rightarrow \chi_{c1} K^0 \)
- \( B^0 \rightarrow \eta_c K^0 \)

- \( N_{\text{sig}} = 8733 \), Purity = 93%

- \( N_{\text{sig}} = 5813 \), Purity = 56%

\[
\sin 2\phi_1 = 0.687 \pm 0.028 \pm 0.012
\]

\[
A_{\text{fCP}} = -0.024 \pm 0.020 \pm 0.016
\]

PRD79,072009(2009)
Now it is a firm SM reference!

\[ \sin(2\beta) \equiv \sin(2\phi_1) \]

**Measurements by B-factories**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>PRD 79 (2009):072009</td>
<td>0.69 ± 0.03 ± 0.01</td>
</tr>
<tr>
<td>BaBar ( \chi_{c0} ) ( K_S )</td>
<td>PRD 80 (2009):112001</td>
<td>0.69 ± 0.52 ± 0.04 ± 0.07</td>
</tr>
<tr>
<td>BaBar ( J/\psi ) (hadronic) ( K_S )</td>
<td>PRD 69 (2004):052001</td>
<td>1.56 ± 0.42 ± 0.21</td>
</tr>
<tr>
<td>Belle</td>
<td>Moriond EW 2011 preliminary</td>
<td>0.67 ± 0.02 ± 0.01</td>
</tr>
</tbody>
</table>

**Measurements before B-factories**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>PLB 492, 259 (2000)</td>
<td>0.84 ± 0.82 ± 0.16</td>
</tr>
<tr>
<td>OPAL</td>
<td>EPJ C5, 379 (1998)</td>
<td>3.20 ± 1.80 ± 0.50</td>
</tr>
<tr>
<td>CDF</td>
<td>PRD 61, 072005 (2000)</td>
<td>0.79 ± 0.41 ± 0.44</td>
</tr>
</tbody>
</table>

**Newcomer, LHCb**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHCb</td>
<td>LHCb-CONF-2011-004</td>
<td>0.53 ± 0.28 ± 0.05</td>
</tr>
<tr>
<td>Average</td>
<td>HFAG</td>
<td>0.68 ± 0.02</td>
</tr>
</tbody>
</table>

(35pb\(^{-1}\), \( A_{CP} = 0 \) assumed.)
b→c\bar{c}d process is pursuit of

In $B^0\to D^+D^-$ case,
If tree (a) dominant, $S_{f\text{CP}} \to -\sin 2\phi_1$, $A_{f\text{CP}} \to 0$,
while if penguin (b) is substantial, complex phase due to $V_{td}$
may cause Direct CPV.

Since $B^0\to D^{(*)+}D^{(*)-}$ is a $B\to VV$ mode, the admixture of
CP even/odd eigenstates must be determined before
measuring CP violation.
$B^0 \rightarrow D^+ D^-$ reconstruction

One $D^+ \rightarrow K^- \pi^+ \pi^+$ (or c.c.), three modes for other $D$.

$D^- \rightarrow K^+ \pi^- \pi^-$ (or c.c.)
$N_{\text{sig}} = 221 \pm 19$

$D^- \rightarrow K_S \pi^-$ (or c.c.)
$N_{\text{sig}} = 48 \pm 9$

$D^- \rightarrow K_S \pi^0 \pi^-$ (or c.c.)
$N_{\text{sig}} = 54 \pm 15$

$\text{Br}(B^0 \rightarrow D^+ D^-) = (2.09 \pm 0.15 \pm 0.18) \times 10^{-4}$

cf. Previous result (PRL98, 221802) based on 535M BB,
$N_{\text{sig}} = 150 \pm 15$ ($D^- \rightarrow K_S \pi^0 \pi^-$ not used), improvement in $N_{\text{sig}}$ by data reprocessing is more significant than (cc) $K^0$ because of the larger track multiplicity.
\[ B^0 \rightarrow D^+D^- \] CP violation

\[ S_{fCP} = -1.06 \pm 0.21 \pm 0.07 \]
\[ A_{fCP} = +0.43 \pm 0.17 \pm 0.04 \]

\( (D^+ \rightarrow K_S \pi^0 \pi^- \) not used because of background forming a peak at same position as signal.\)

\( S_{fCP} \) is similar. However \( A_{fCP} \) has decreased compared to previous publication with 535M \( B\bar{B} \) (PRL98,221802).

\( (S_{fCP} = -1.13 \pm 0.37 \pm 0.09, A_{fCP} = +0.91 \pm 0.23 \pm 0.06) \)
$B^0 \rightarrow D^{*+}D^{*-}$ branching and polarization

$N_{\text{sig}} = 1225 \pm 59$

( was 553$\pm$30 for 657M BB, PRD80,111104.)

$\text{Br}(B^0 \rightarrow D^{*+}D^{*-}) = (7.82 \pm 0.38 \pm 0.60) \times 10^{-4}$

$R_0 = 0.62 \pm 0.03 \pm 0.01$ (longitudinal pol.)

$R_{\text{perp}} = 0.14 \pm 0.02 \pm 0.01$ (CP-odd)
As a result of data reprocessing, signal yield from 772M $B\bar{B}$ pairs is $\times 2.2$ larger than the yield with the 657M $B\bar{B}$ sample used for the previous result (PRD80,111104).

→ significant improvement ($S_{fCP}$ and $A_{fCP}$ errors down to 60%)!
There are 3 modes; $\pi\pi$, $\rho\rho$, $\rho\pi$. In addition $a_1\pi$. 

$\phi_2$ measurement

If tree only, $S_f$ is directly connected to $\sin2\phi_2$ and $A_f=0$.

Interference with $b \rightarrow d$ penguin can be solved by isospin analysis.

Mixing diagram

Decay diagram (tree)
Extract $\phi_2$; isospin analysis

M. Gronau and D. London, PRL 65, 3381 (1990)

\[ \frac{1}{\sqrt{2}} \tilde{A}^{-} \]
\[ \frac{1}{\sqrt{2}} \tilde{A}^{+} \]
\[ 2\theta \]
\[ A^{+0} = \tilde{A}^{00} \]

\[ A^{+0} (\tilde{A}^{00}) \]
\[ B^{0} (\bar{B}^{0}) \rightarrow \pi^{+} \pi^{-} \]
\[ A^{00} (\tilde{A}^{00}) \]
\[ B^{0} (\bar{B}^{0}) \rightarrow \pi^{0} \pi^{0} \]
\[ A^{+0} (\tilde{A}^{-0}) \]
\[ B^{+} (\bar{B}^{-}) \rightarrow \pi^{+} \pi^{0} (\pi^{-} \pi^{0}) \]

\[ \tilde{A}^{ij} = e^{2\phi} \bar{A}^{ij} \]

$B^{0} \rightarrow \pi^{+} \pi^{-}$, $\pi^{0} \pi^{0}$, $B^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ branching fractions, and $B^{0} \rightarrow \pi^{0} \pi^{0}$ Direct CPV are used as inputs to solve this relation. The correction from SU(2) breaking effect is still much smaller than measurements’ errors.
\[ B^0 \rightarrow \rho^+ \rho^- \]

B → VV, almost purely longitudinally polarized = CP eigenstate. Small Br(B^0 → \rho^0\rho^0), i.e. small penguin pollution.

\[ f_L = 0.941 \pm 0.034/-0.040 \pm 0.030 \]

\[ S_f = 0.19 \pm 0.30 \pm 0.07 \]

\[ A_f = 0.16 \pm 0.21 \pm 0.07 \]
Constraint on $\phi_2$

$\phi_2 = 89.0 \pm 4.4/-4.2$ deg.
As for $\Delta m_d$ measurement

BaBar: $D^*\ell\nu$ partial recon., opposite side B is tagged by high momentum lepton.

Belle: $D^*\ell\nu$ and $D^{(*)}X$ hadronic modes full recon., opposite side B tagging is the one for time-dependent CPV.

$\Delta m_d$ and B lifetime are obtained simultaneously. With $\sim 20\%$ of entire $\Upsilon(4S)$ data, but systematic dominant.
$\Delta m_d$ without/with B-factories

BaBar and Belle results 4-5 times more precise than LEP and Tevatron experiments.

Now 1% precision has been achieved. This gives another reference point to constrain unitarity triangle, i.e. $|V_{td}|$ in the SM framework.
KM scheme has been tested.

\( \phi_3 \) precision improved, \( \sigma(\phi_3) \sim 10^\circ \) (See Y.Horii’s talk).

Is the unitarity triangle a right triangle?
However, tension with $\text{Br}(B^+ \rightarrow \tau^+ \nu)$

Of course, new physics (NP) could be present in $B^+ \rightarrow \tau^+ \nu$. We need an update of this measurement. But no other place?

$\sin 2\phi_1$ measurement gives a stringent constraint.
\( S_{\text{fCP}} \) and \( \sin 2\phi_1 \) SM relation

(1) Decay

\[
\begin{align*}
B^0 & \rightarrow c \bar{c} J/\psi \\
\bar{d} & \rightarrow s \bar{d} K^0
\end{align*}
\]

(2) Decay with mixing

\[
\begin{align*}
B^0 & \rightarrow t \bar{t} V_{td}^* \\
\bar{d} & \rightarrow s \bar{d} K^0
\end{align*}
\]

Interference between (1) and (2) results in CP violation.

\( S_{\text{fCP}} = -\xi_{\text{CP}} \sin 2\phi_1, \quad \xi_{\text{CP}} = -1 \) (CP-odd), \( +1 \) (CP-even), \( A_{\text{fCP}} = 0 \).

Is there room to accommodate new physics (NP)?
NP room is unlikely in $b \to c\bar{c}s$ decays

If NP penguin is substantial and has different phase, it causes Direct CPV in $B^\pm \to J/\psi K^\pm$.
No direct CP violation has been observed so far.
-0.76±0.50±0.22% at Belle (PRD82,091104),
$(1\pm7)\times10^{-3}$ in PDG2011.

However, there is room for NP in B-B mixing.
Effective $\phi_1$ in penguin decays

SM penguin; No complex phase in decay.

as well as

New Physics in the loop; may have a different weak phase.
CPV deviation from $J/\psi K^0$?

Many two-body and quasi-two body analyses have been done. Since $\phi \rightarrow K^+K^-$, $f_0 \rightarrow K^+K^-$ and non-resonant contributions overlap in invariant mass (as do $\rho^0 \rightarrow \pi^+\pi^-$ and $f_0 \rightarrow \pi^+\pi^-$), recently time-dependent Dalitz analyses have been performed in three-body decays such as $B^0 \rightarrow (K^+K^-)K_S$ and $B^0 \rightarrow (\pi^+\pi^-)K_S$. 
Several contributions are overlapping

- For example, $B^0 \rightarrow K^+ K^- K_S$ final state has several different paths.
- Resolve them by fitting the Dalitz distribution. Same approach is required for $B^0 \rightarrow \pi^+ \pi^- K_S$. 

\[ \phi_{K_S^0} = \begin{cases} -1 & \text{for } f_0(980)K_S^0 \\ +1 & \text{for } \phi_{K_S^0} \end{cases} \]
Projections of Dalitz distribution ($M_{K^+K^-}$)

Peak around 1GeV/c² : $\phi(1020)$ and $f_0(980)$, at 1.5GeV/c² : $fX$, at 3.4GeV/c² : $\chi_{c0}$

There are multiple solutions (Belle found 4, BaBar found 2).
$\Delta t$ distribution in $\phi$ mass region

PRD82,073011(2010)

arXiv:0808.0700
Effective $\phi_1$ of “solution 1”

<table>
<thead>
<tr>
<th>$K^+$</th>
<th>$K^-$</th>
<th>$K_S$</th>
<th>$\beta(\phi K_S)$</th>
<th>$\beta(f_0 K_S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BaBar</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>arXiv:0808.0700</td>
<td></td>
<td></td>
<td>$7.7 \pm 7.7 \pm 0.9$</td>
<td>$8.5 \pm 7.5 \pm 1.8$</td>
</tr>
<tr>
<td><strong>Belle</strong></td>
<td></td>
<td></td>
<td>$32.2 \pm 9.0 \pm 2.6 \pm 1.4$</td>
<td>$31.3 \pm 9.0 \pm 3.4 \pm 4.0$</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td>$16.9 \pm 6.0$</td>
<td>$16.9 \pm 6.1$</td>
</tr>
</tbody>
</table>

No significant deviation from measurements with $B^0 \rightarrow (c\bar{c}) K^0$. 

With current statistics, we could not distinguish multiple solutions by the likelihood alone. The preferred solution is shown.
Compilation of effective $\sin 2\phi_1$

$$\sin(2\beta^{\text{eff}}) = \sin(2\phi_1^{\text{eff}})$$

Still precision is statistically dominated.

To obtain sensitivity in effective $\sin 2\phi_1$ of $O(10^{-2})$, we need $O(10\text{ab}^{-1})$ integrated luminosity.
Future sensitivity

Error of effective $\sin 2\phi_1$ would be $0.03(\eta'K^0) - 0.1(K_SK_SK_S)$. 

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Summary

- $\sin^2 \phi_1 = 0.68 \pm 0.02$ in World Average
  - It is a firm SM reference point.
- Constraint on $\phi_2$ : $89.0^{+4.4/-4.2}$ deg.
  - The unitarity triangle appears to be a right triangle.
- $\Delta m_d$ is precisely determined by B-factories.
  - Now 1% precision has been achieved, giving a firm reference.
- Tension around $\text{Br}(B^+ \rightarrow \tau^+ \nu)$
  - Need an update of measurement.
  - Comparing to $\sin^2 \phi_1$ measurement, expect mixing has room for NP.
- CPV in $b \rightarrow s$ penguin modes
  - Reach $O(10^{-2})$ sensitivity with Super B-factories.
Backup slides
$B^0 \rightarrow \pi^+ \pi^-$

**Graph (a):**
- Belle
- $q = +1$ (solid line)
- $q = -1$ (dashed line)
- $N_{\text{sig}} = 1464$

**Graph (b):**
- $A_{CP}$
- $\Delta t$ (ps)

$S_f = -0.61 \pm 0.10 \pm 0.04$
$A_f = 0.55 \pm 0.08 \pm 0.05$

**Graph (c):**
- BaBar
- $B^0(\Delta t)$
- $N_{\text{sig}} = 1394$

$S_f = -0.68 \pm 0.10 \pm 0.03$
$A_f = 0.25 \pm 0.08 \pm 0.02$
$B^0 \rightarrow (\rho \pi)^0, \ B^0 \rightarrow a_1^{\pm} \pi^{\mp}$

$B^0 \rightarrow \rho^+ \pi^-, \rho^- \pi^+, \rho^0 \pi^0$ interfere
→ time-dep. Dalitz analysis

BaBar (384M BB) $a_1^{\pm} \pi^{\mp}$; obtained $\alpha^{\text{eff}} (= \phi_2^{\text{eff}})$

Belle

$N_{\text{sig}} = 971$

PRL98, 221602 (2007)

BaBar (384M BB)

PRD76, 012004 (2007)

PRL98, 181803 (2007)
Coefficients of Dalitz plot functions are interrupted to CPV parameters of quasi-2-body decays, $B \rightarrow \rho^{+}\pi^{-}$ and $B \rightarrow \rho^{0}\pi^{0}$

$\begin{align*}
    c^{+} &= \frac{U_{-}}{U_{+}}, \quad c^{-} = \frac{U_{-}}{U_{+}}, \quad s^{+} = \frac{2I^{+}}{U_{+}}, \quad s^{-} = \frac{2I^{-}}{U_{+}}, \quad A_{\rho^{+}\pi^{-}}^{CP} = \frac{U_{+} - U_{-}}{U_{+} + U_{-}} \\
    c &= \frac{c^{+} + c^{-}}{2}, \quad \Delta c &= \frac{c^{+} - c^{-}}{2}, \quad s &= \frac{s^{+} + s^{-}}{2}, \quad \Delta s &= \frac{s^{+} - s^{-}}{2}
\end{align*}$

Belle 449M $B\bar{B}$ (PRL98 221602)

$\begin{align*}
    A_{\rho^{+}\pi^{-}}^{CP} &= -0.12 \pm 0.05 \pm 0.04 \\
    C &= -0.13 \pm 0.09 \pm 0.05 \\
    \Delta C &= +0.36 \pm 0.10 \pm 0.05 \\
    S &= +0.06 \pm 0.13 \pm 0.05 \\
    \Delta S &= -0.08 \pm 0.13 \pm 0.05 \\
    A_{\rho^{0}\pi^{0}} &= -0.49 \pm 0.36 \pm 0.28 \\
    S_{\rho^{0}\pi^{0}} &= +0.17 \pm 0.57 \pm 0.35
\end{align*}$

BABAR 375M $B\bar{B}$ (PRD76 012004)

$\begin{align*}
    A_{\rho^{+}\pi^{-}} &= -0.14 \pm 0.05 \pm 0.02 \\
    C &= 0.15 \pm 0.09 \pm 0.05 \\
    S &= -0.03 \pm 0.11 \pm 0.04 \\
    \Delta C &= 0.39 \pm 0.09 \pm 0.09 \\
    \Delta S &= -0.01 \pm 0.14 \pm 0.06 \\
    C_{00} &= \frac{U_{-}}{U_{0}} = -0.10 \pm 0.40 \pm 0.53 \\
    S_{00} &= \frac{2I_{0}}{U_{+}} = 0.04 \pm 0.44 \pm 0.18
\end{align*}$

$68^\circ < \phi_{2} < 95^\circ$ (68.3% C.L.)

$\phi_{2} = (87^{+45}_{-13})^\circ$ (68.3% C.L.)
Belle 449M $B\bar{B}$ (PRL98 221602)

971±41 signal yields

$B^0 \to \rho^+ \pi^-$

$B^0 \to \rho^- \pi^+$

$B^0 \to \rho^0 \pi^0$

BABAR 375M $B\bar{B}$ (PRD76 012004)

2067±86 signal yields
Multiple solutions

Belle found 4 solutions

<table>
<thead>
<tr>
<th>Name</th>
<th>Solution (1)</th>
<th>Solution (2)</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{CP}(f_0(980)K_S^0)$</td>
<td>$0.14 \pm 0.19 \pm 0.02$</td>
<td>$0.13 \pm 0.18$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\phi_{\text{eff}} (f_0(980)K_S^0)$</td>
<td>$0.13 \pm 0.13 \pm 0.02$</td>
<td>$0.14 \pm 0.14$</td>
<td>-0.09</td>
</tr>
<tr>
<td>$A_{CP}(\phi(1020)K_S^0)$</td>
<td>$0.01 \pm 0.26 \pm 0.07$</td>
<td>$-0.49 \pm 0.25$</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\phi_{\text{eff}} (\phi(1020)K_S^0)$</td>
<td>$0.15 \pm 0.13 \pm 0.03$</td>
<td>$3.44 \pm 0.19$</td>
<td>0.09</td>
</tr>
</tbody>
</table>

The preferred solution cannot be selected by the fit likelihood value alone. With external information, solution 1 is preferred.

BaBar found 2 solutions in low-mass fit, (1) is chosen as nominal.
Belle found 4 solutions. After ensemble test checks and by using external information, two of them are chosen as possible physical solutions. Solution 1 is preferred ($K^*_0(1430)\pi$ fraction and $K_S\pi$ mass spectrum).  

(PRD79,072004(2009))

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution 1 (-2ln$L$=18472.5)</th>
<th>Solution 2 (-2ln$L$=18465.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(f_0K_S)$</td>
<td>0.08±0.19±0.03±0.04</td>
<td>0.23±0.19±0.03±0.04</td>
</tr>
<tr>
<td>$\beta(f_0K_S)=\phi_1(f_0K_S)$</td>
<td>(36.0±9.8±2.1±2.1)$^\circ$</td>
<td>(56.2±10.4±2.1±2.1)$^\circ$</td>
</tr>
<tr>
<td>$A(\rho^0K_S)$</td>
<td>-0.05±0.26±0.10±0.03</td>
<td>-0.14±0.26±0.10±0.03</td>
</tr>
<tr>
<td>$\beta(\rho^0K_S)=\phi_1(\rho^0K_S)$</td>
<td>(10.2±8.9±3.0±1.9)$^\circ$</td>
<td>(33.4±10.4±3.0±1.9)$^\circ$</td>
</tr>
</tbody>
</table>
Again multiple solutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution 1</th>
<th>Solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(f_0 K_S) = -A(f_0 K_S)$</td>
<td>$0.08 \pm 0.19 \pm 0.03 \pm 0.04$</td>
<td>$0.23 \pm 0.19 \pm 0.03 \pm 0.04$</td>
</tr>
<tr>
<td>$\beta(f_0 K_S) = \phi_1(f_0 K_S)$</td>
<td>$(36.0 \pm 9.8 \pm 2.1 \pm 2.1)^\circ$</td>
<td>$(56.2 \pm 10.4 \pm 2.1 \pm 2.1)^\circ$</td>
</tr>
<tr>
<td>$C(\rho^0 K_S) = -A(\rho^0 K_S)$</td>
<td>$-0.05 \pm 0.26 \pm 0.10 \pm 0.03$</td>
<td>$-0.14 \pm 0.26 \pm 0.10 \pm 0.03$</td>
</tr>
<tr>
<td>$\beta(\rho^0 K_S) = \phi_1(\rho^0 K_S)$</td>
<td>$(10.2 \pm 8.9 \pm 3.0 \pm 1.9)^\circ$</td>
<td>$(33.4 \pm 10.4 \pm 3.0 \pm 1.9)^\circ$</td>
</tr>
</tbody>
</table>

BaBar found 2 solutions. (PRD80,112001(2009))