Measurements of the Unitarity Triangle Sides

$|V_{ub}|$ and $|V_{cb}|$ from Semileptonic B decays

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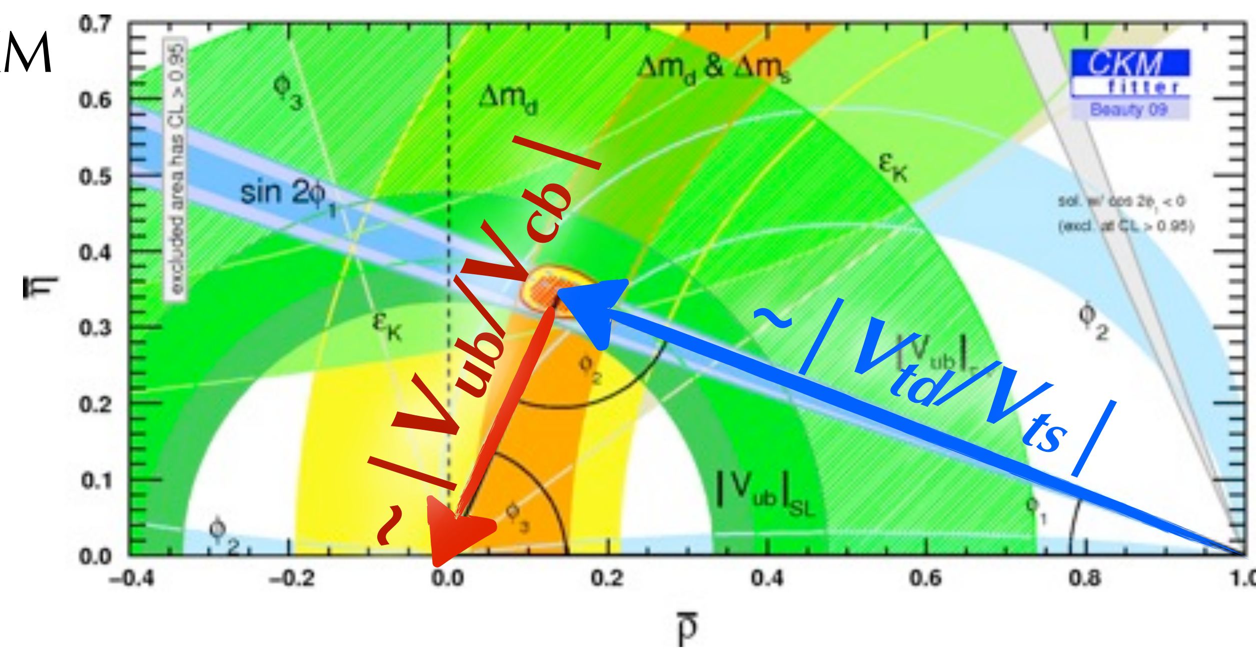
Introduction

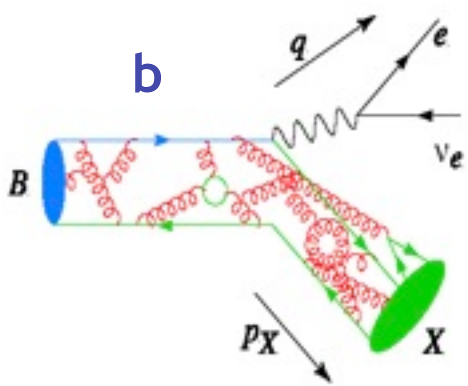
CKM matrix elements are fundamental parameters of the Standard Model and cannot be predicted.

Exploit the unitarity constraint to look for new physics → geometrical relation between CKM elements: angle from CP asymmetries, size from $|V_{CKM}|$.

Precision era: new physics may appear as a few percent disagreement: large new physics contributions to penguins would have already been seen.



We must make the green ring thinner → uncertainty dominated by |Vub|



Inclusive $|V_{cb}|$: $B \to X_c \, l \, \nu$

Exclusive $|V_{cb}|$: $B \to D^{(*)} \, l \, \nu$

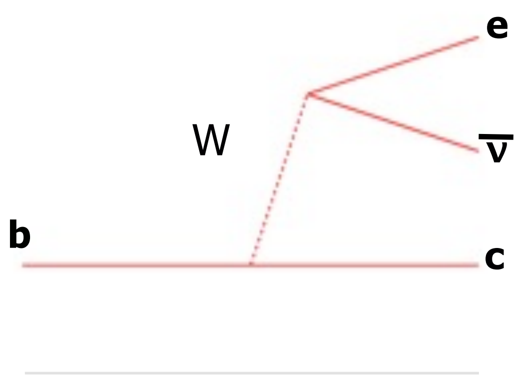
Inclusive $|V_{ub}|$: $B \to X_u \, l \, \nu$

Exclusive $|V_{ub}|$: $B \to \pi \, l \, \nu$

Semileptonic B decays

tree level, short distance:

$b \rightarrow c\, e\, \nu$



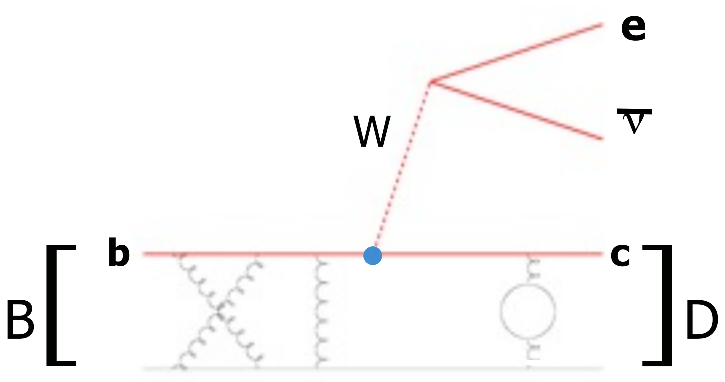
Decay properties depend directly on $|V_{cb}|$ & $|V_{ub}|$ and m_b

perturbative regime (α_s^n).

Semileptonic B decays

tree level, short distance:

$B \to D\, e\, \nu$



+ long distance:

Decay properties depend directly on $|V_{cb}|$ & $|V_{ub}|$ and m_b

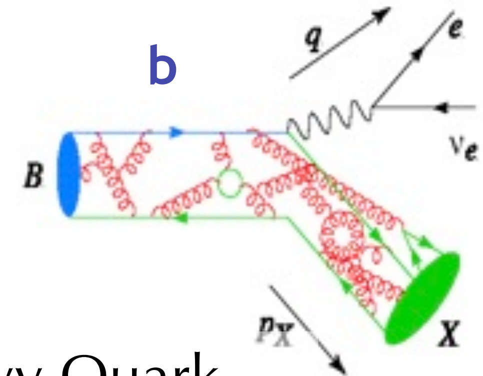
perturbative regime (α_s^n).

But quarks are bound by soft gluons: non-perturbative

long distance interactions of *b* quark with light quark.

Exclusive Vs. Inclusive

One hadronic current.



Inclusive decays b→q l ν:

Weak quark decay + QCD corrections.

Γ_{sl} described by Heavy Quark Expansion in $(1/m_b)^n$ and α_s^k

$$\Gamma(B \to X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 [[1 + A_{ew}] A_{nonpert} A_{pert}]$$

Non perturbative parameters need to be derived from data, i.e. from inclusive spectral moments of the semileptonic decay products.

Theoretically easier, more precise.

Exclusive decays B→X_q l ν:

Form factors: need lattice QCD.

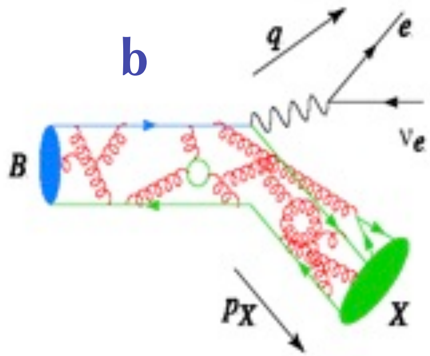
$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2}{24\pi^2} |V_{ub}|^2 p_\pi^3 |f_+(q^2)|^2$$

i.e. Currently use B → π l ν for $|V_{ub}|$ - one dominant form factor (q^2 shape and normalization needed).

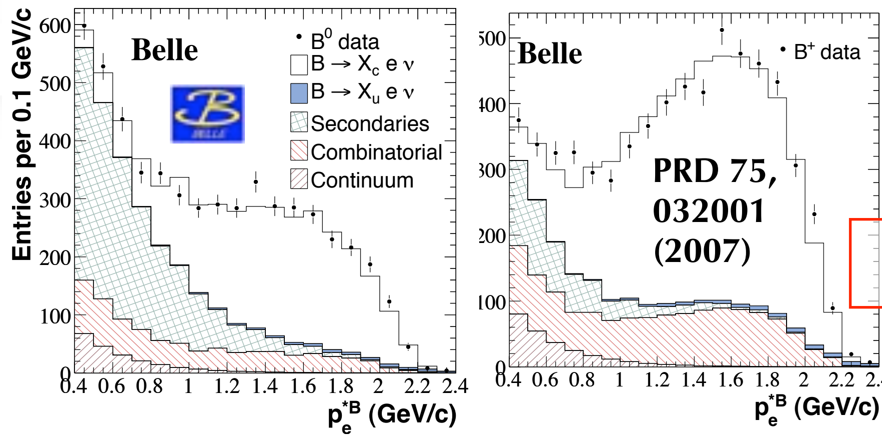
Experimentally clean, a check of inclusive methods.

Inclusive decays: Big Picture

Semileptonic *B* decay



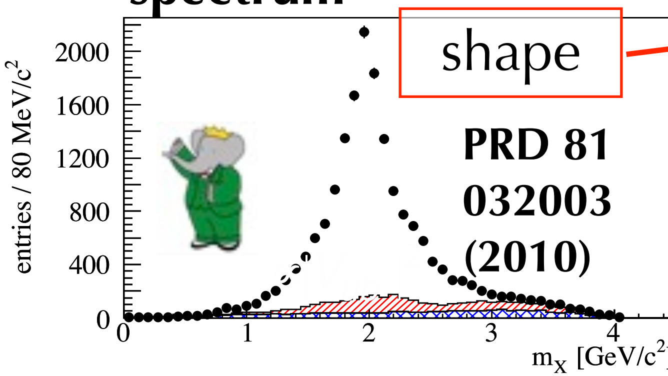
Inclusive E_l spectrum



rate → $|V_{cb}|$

shape → m_b, m_c, μ^2_G, μ^2_π

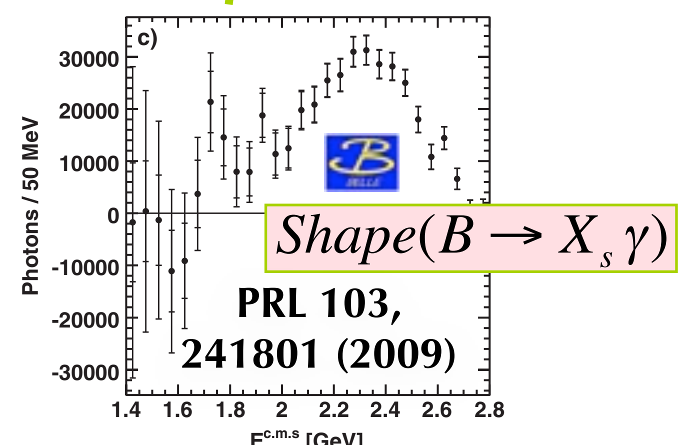
Inclusive M_X spectrum



shape → m_b, m_c, μ^2_G, μ^2_π

m_b, m_c, μ^2_G, μ^2_π → $|V_{cb}|$

m_b → $|V_{ub}|$



Experimental Challenge:

Go from the measured shape → true shape:

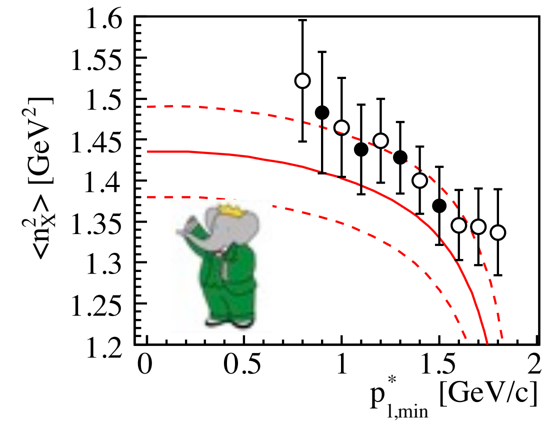
shape in B rest frame, QED corrections, detector resolution, accessible phase space, $X_c l \nu$ model etc.

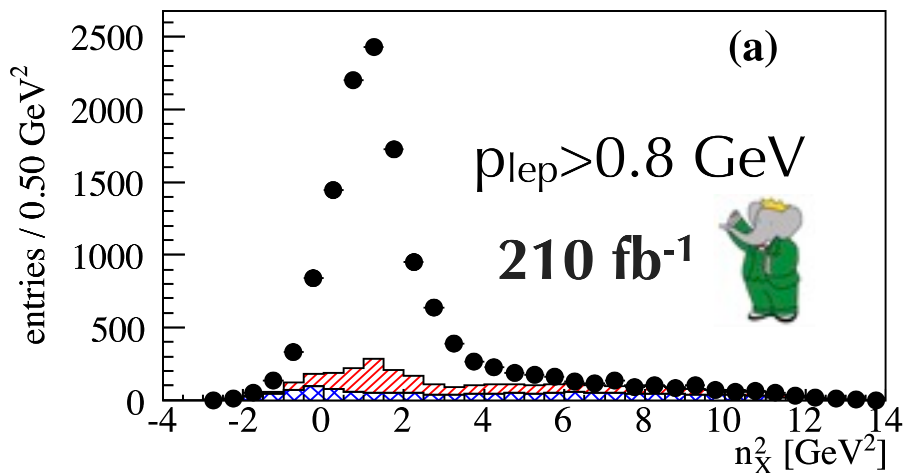
Moments from Babar

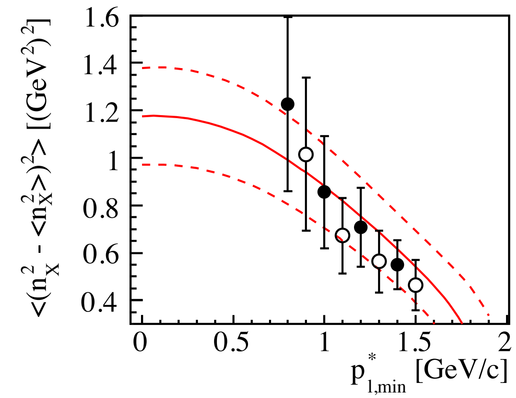
Recent moments results from Babar also include "mixed" moments **PRD 81 032003 (2010)**

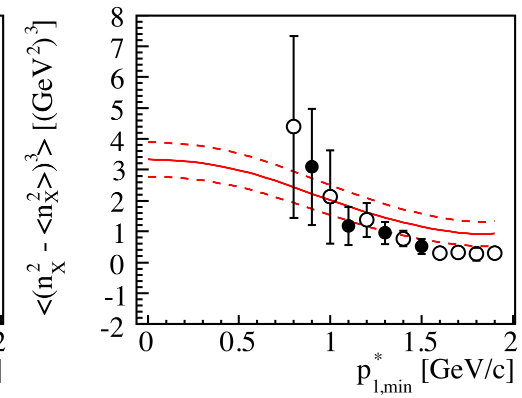
Alternative extraction of the higher-order nonperturbative HQE parameters

$\langle n_X^2 \rangle^k$:
$n_X^2 = M_X^2 - 2\Lambda E_X + \Lambda^2$









Kinetic scheme	Mass moments	Mixed moments	Belle 2008 PRD78 032016 (2008)
$\vert V_{cb}\vert 10^3$	42.05±0.83	41.91±0.85	41.58±0.90
m_b^{kin}[GeV]	4.549±0.049	4.566±0.053	4.543±0.075

Different experiments in good agreement: confidence in OPE fits.

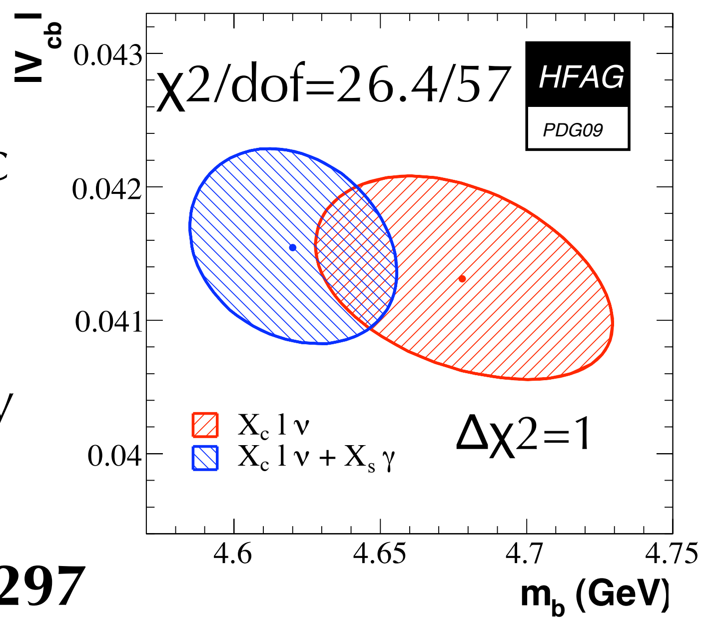
$|V_{cb}|$ from Global Fit

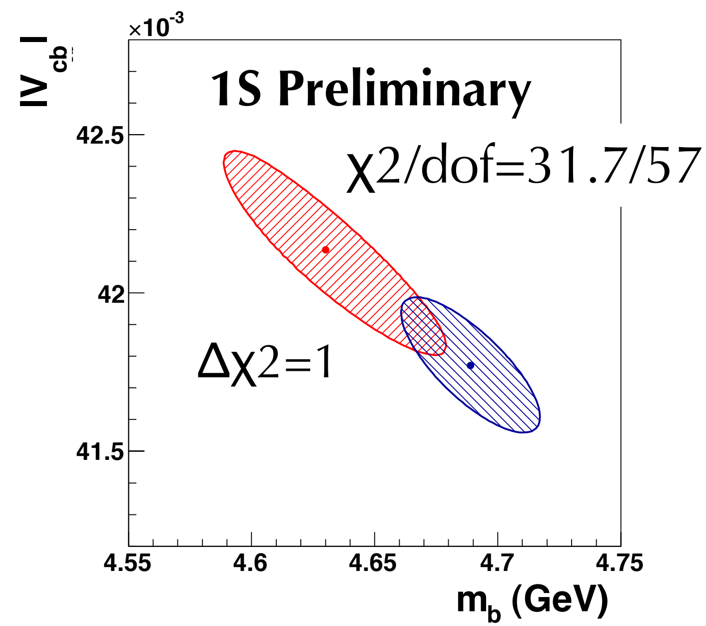
HFAG averages different measurements in the Kinetic and 1S schemes:
27 from Babar, 25 from Belle, 12 from CDF+CLEO+DELPHI.

	Kinetic: EPJ C34, 181 (2004)		1S: PRD70, 094017 (2004)	
S.L. +Rad.	$\|V_{cb}\| 10^3$	41.54±0.43±0.08±0.58	$\|V_{cb}\| 10^3$	41.77±0.21±0.08
	m_b^{kin}[GeV]	4.620±0.035	m_b^{1S}[GeV]	4.689±0.028
	μ_π^2[GeV^2]	0.440±0.040	λ_1[GeV^2]	-0.336±0.022
S.L.	$\|V_{cb}\| 10^3$	41.31±0.49±0.08±0.58	$\|V_{cb}\| 10^3$	42.14±0.33±0.08
	m_b^{kin} [GeV]	4.678±0.051	m_b^{1S}[GeV]	4.630±0.047
	μ_π^2 [GeV^2]	0.428±0.044	λ_1[GeV^2]	-0.377±0.031

Excellent agreement between 1S & kinetic scheme.

δ|Vcb|/|Vcb| ~1-2% dominated by theory uncertainties.





arXiv:0808.1297

Exclusive decays: B→D$^{(*)}$l ν

Differential decay rate:

$$\frac{d\Gamma}{dw}(B \to D\ell\nu) \sim (\text{Phase Space})|V_{cb}|^2 G(w)^2$$

$$\frac{d\Gamma}{dw}(B \to D^*\ell\nu) \sim (\text{Phase Space})|V_{cb}|^2 F(w)^2 \sum_{i=+,0,-} |H_i(w)|^2$$

$$w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_d}$$

Form factors can be parameterised:

$$G(w) = G(1)[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3], \ z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w-1} + \sqrt{2}}$$

$$F(w) = \ldots$$

From experiment

|Vcb| x F.F. @w=1 (0 recoil)

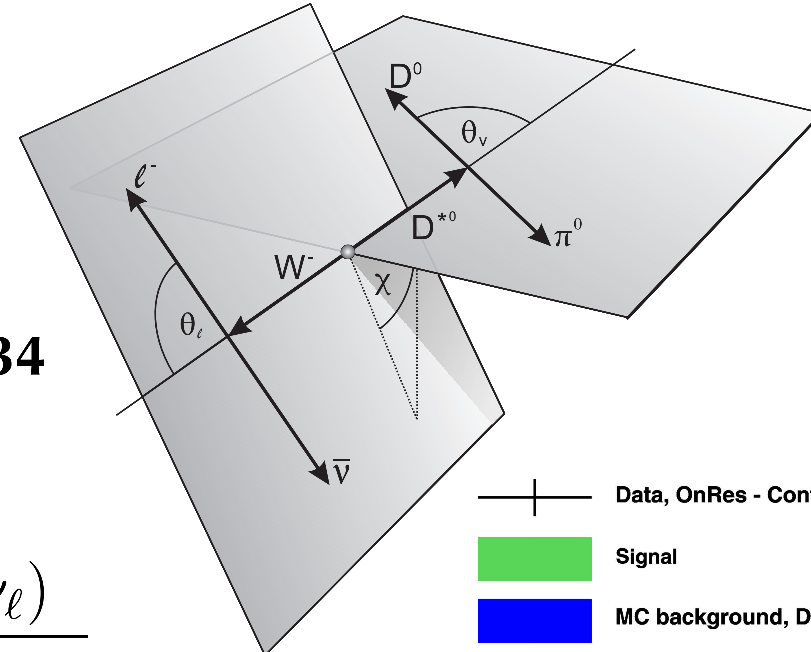
ρD, ρD* (F.F. slopes)

From Lattice

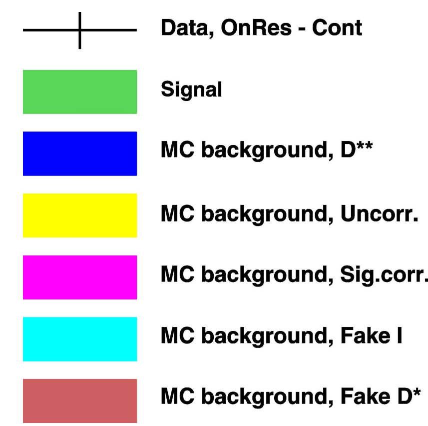
G(1)=1.074±0.024, **NPPS 140, 461 (2005)**

F(1)=0.921±0.024, **PRD 79 014506 (2009)**

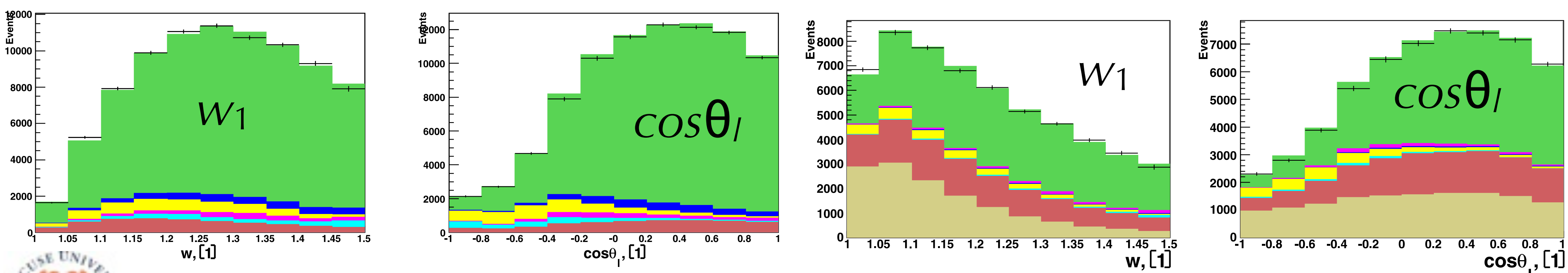
$B \rightarrow D^{*}l\,\nu$ from Belle



- Study charged and neutral B decays:
  - $B^0 \rightarrow D^{*-}l^+\,\nu$, $D^* \rightarrow D^0\pi^-$ **arXiv:0810.1657**
  - $B^\pm \rightarrow D^{*0}l^+\,\nu$, $D^{0*} \rightarrow D^0\pi^0$ **arXiv:0910.3534**
- Measure w and decay angles θ_ℓ, θ_v , χ
- Fit 4-D decay rate $$\frac{d^4\Gamma(B^+ \rightarrow \bar{D}^{*0}\ell^+\nu_\ell)}{dw\,d(\cos\theta_\ell)d(\cos\theta_V)d\chi}$$



$B^0 \rightarrow D^{*-}l^+\,\nu$ **140 fb^{-1}** $B^\pm \rightarrow D^{*0}l^+\,\nu$



B→D*l ν from Belle

Results of 4-parameter HQET parameterization fit.

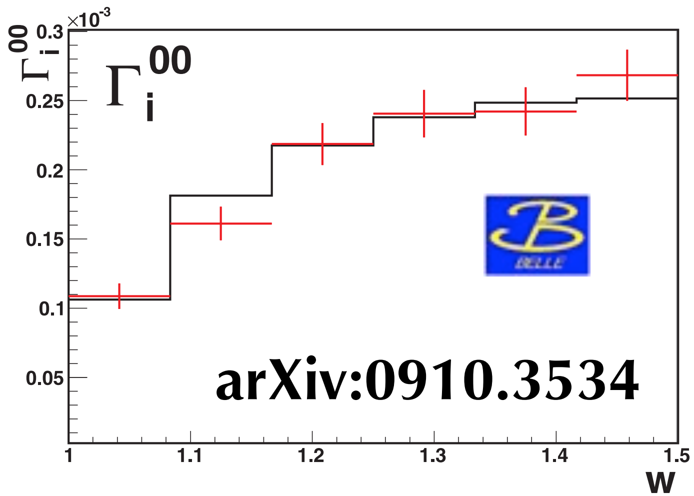
B+/B0 are consistent.

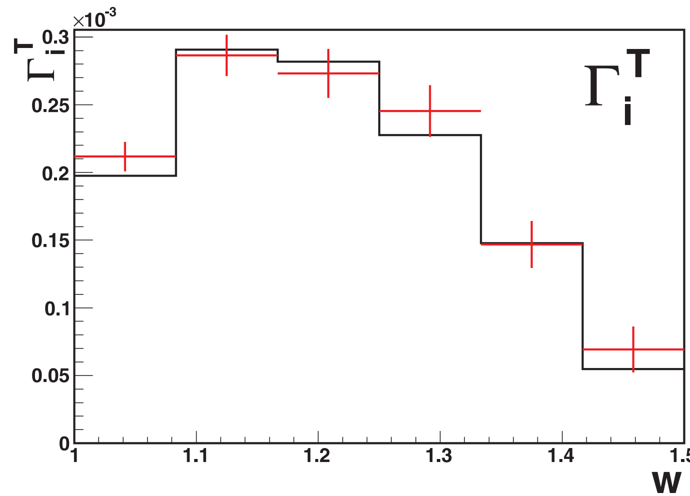
Relatively low values of F(1)|Vcb|.

	$B^0 \to D^{*-} l^+ \nu$ arXiv:0810.1657	$B^\pm \to D^{*0} l^+ \nu$ arXiv:0910.3534
ρ^2	1.293±0.045±0.029	1.376±0.074±0.056
$R_1(1)$	1.495±0.050±0.062	1.620±0.091±0.092
$R_2(1)$	0.844±0.034±0.019	0.804±0.064±0.036
BR(%)	4.42±0.03±0.25	4.84±0.04±0.56
$F(1)\|V_{cb}\|$	34.4±0.2±1.0	35.0±0.4±2.2
χ^2/dof	138.8/155	187.8/155

Belle performs a **model independent measurement** of F.F. shapes.

Confirms use of Caprini et al. parameterisation.



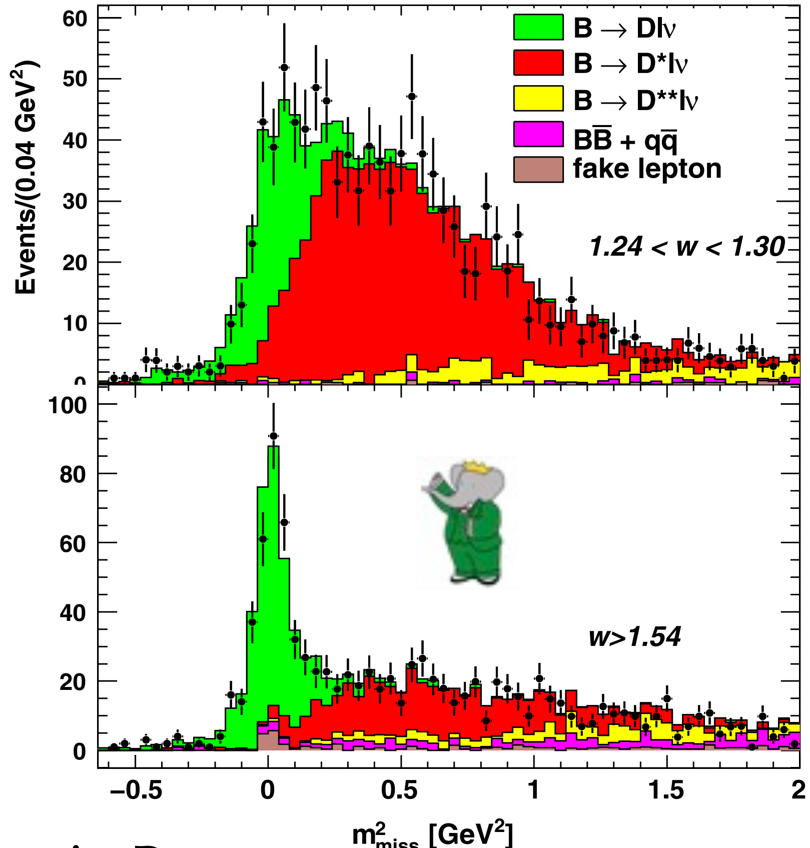


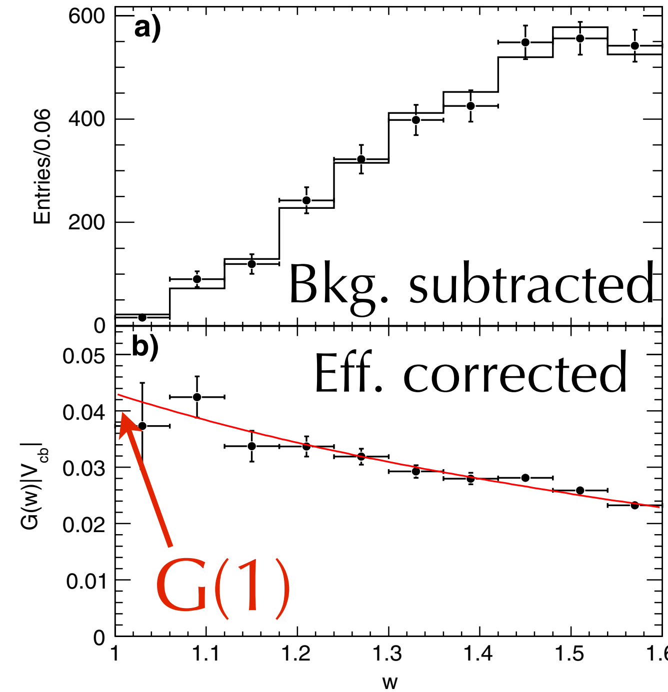
B→D$^{(*)}$l ν from Babar

Two recent, complementary, B→D l ν results from Babar.

- **Untagged**, simultaneous fit of B→D*l ν and B→Dl ν, **PRD 79, 012002 (2009)**

- **Hadronic B-tag** measurement, **PRL 104 011802 (2010)**





Hadronic B-tag

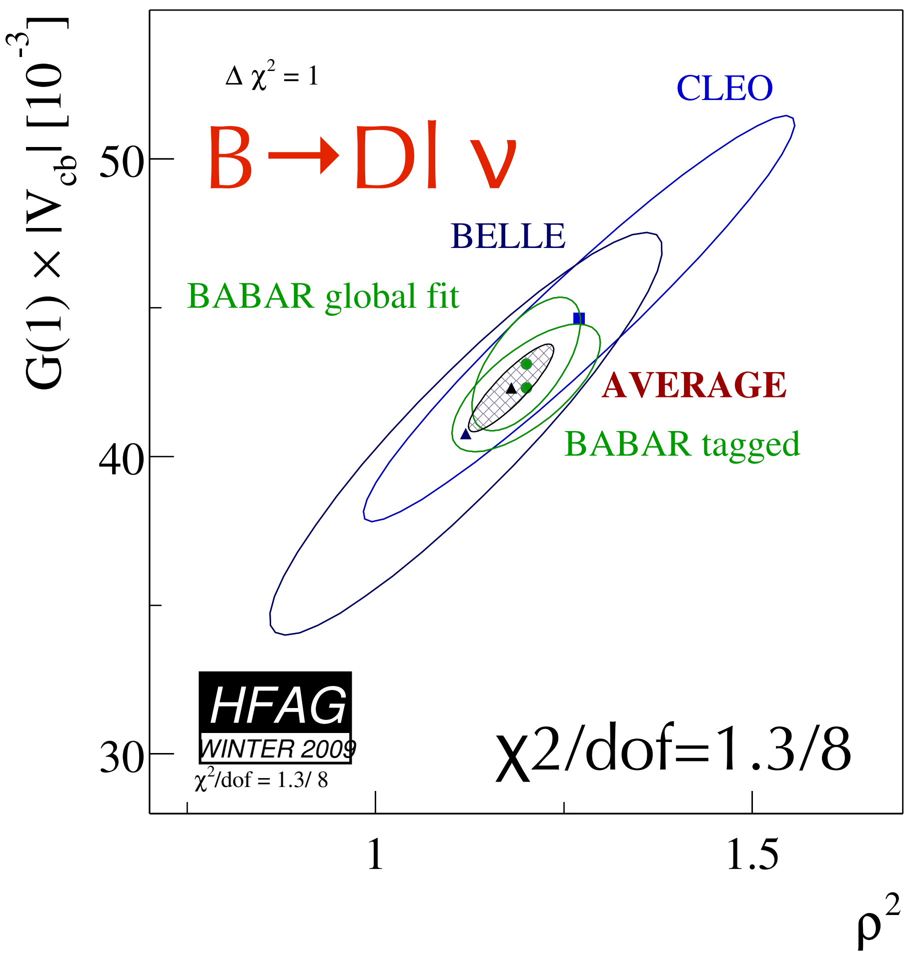
417 fb^{-1}

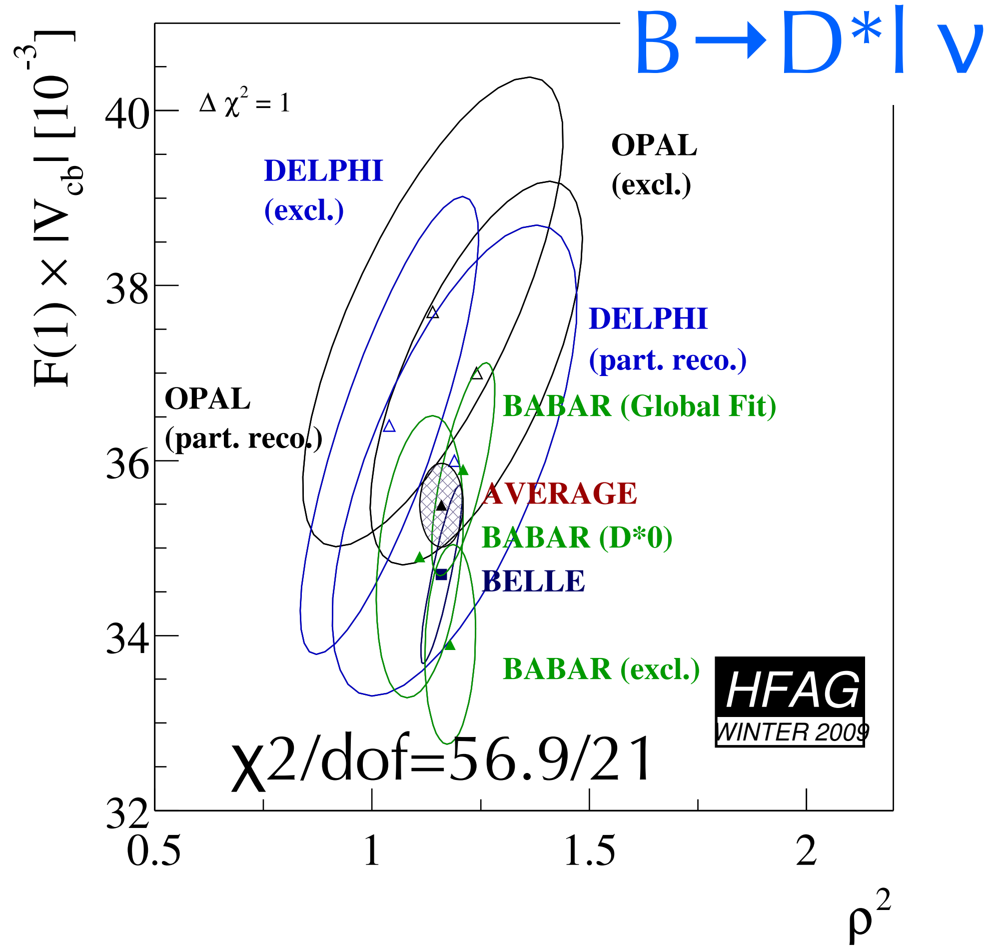
$|V_{cb}|G(1) = (42.3 \pm 1.9 \pm 1.4)\ 10^{-3}$

ρ_D^2=1.20±0.09±0.04

BR(B-→D l ν)=(2.15±0.06±0.09)%

$|V_{cb}|$ from $B \to D^{(*)} l \nu$





$|V_{cb}|G(1) = (42.3 \pm 0.7 \pm 1.3)\ 10^{-3}$

⇨ $|V_{cb}| = (39.4 \pm 1.4 \pm 0.9(FF))\ 10^{-3}$

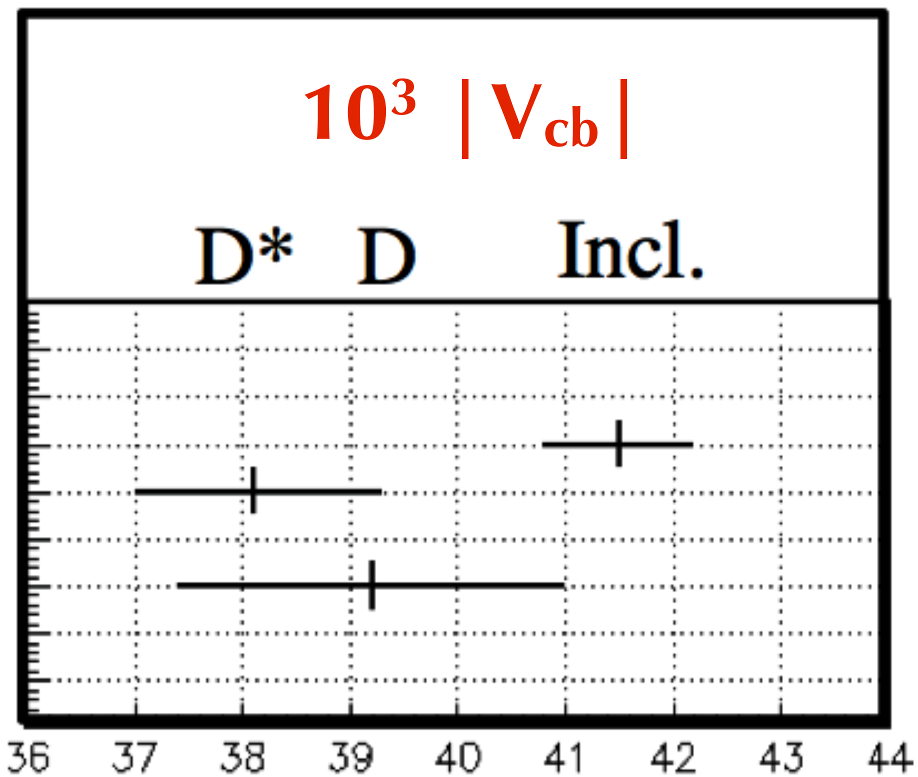
precision ~4%

$|V_{cb}|F(1) = (35.75 \pm 0.42)\ 10^{-3}$

⇨ $|V_{cb}| = (38.8 \pm 0.5 \pm 1.0(FF))\ 10^{-3}$

precision ~3%, tension in ave.

$|V_{cb}|$ summary: Inclusive v Exclusive



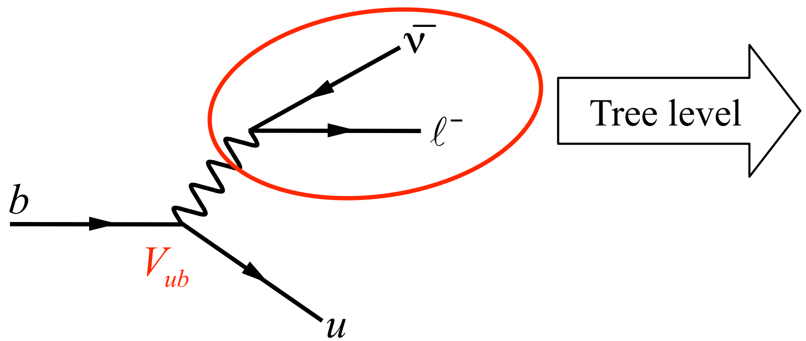
HFAG averages

Exclusive |Vcb| ~2σ lower than inclusive

$|V_{ub}|$ Challenge

Limiting factor in CKM precision tests; known much less well than $|V_{cb}|$

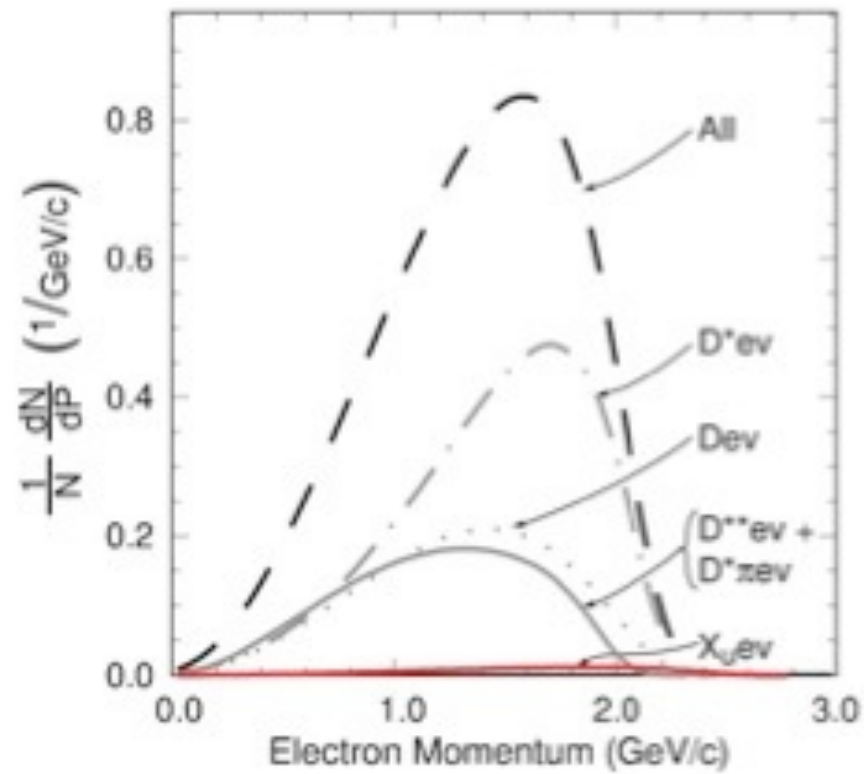
CKM suppressed $V_{ub} \sim 0.1 x V_{cb}$ - therefore harder to measure.



$$\Gamma(b \to u \ell^- \bar{\nu}) = \frac{G_F^2}{192\pi^2} |V_{ub}|^2 m_b^5$$

The problem: *b* → *clv* decay

$$\frac{\Gamma(b \to u \ell \bar{\nu})}{\Gamma(b \to c \ell \bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$



Inclusive $|V_{ub}|$ Measurement

- Cut away $b \to c\ell\nu$: lose a part of the $b \to u\ell\nu$ signal.

  - We measure $$\Gamma(B \to X_u \ell \nu) \times f_C = |V_{ub}|^2 \zeta_C$$

Total $b \to u\ell\nu$ rate

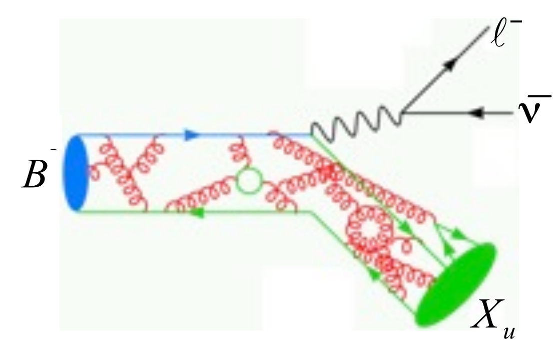
Cut-dependent constant predicted by theory

Fraction of the signal that pass the cut

→ corrected for QCD, motion of b-quark

$f_C \sim 25\%$ for $E_\ell > 2.0$ GeV,
$f_C \sim 38\%$ for $q^2 > 8$ GeV^2,
$f_C \sim 65\%$ for $M_X < 1.7$ GeV

$$\Gamma(B \to X_u \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3}\left[1 - O\left(\frac{\alpha_s}{\pi}\right) - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \cdots\right]$$



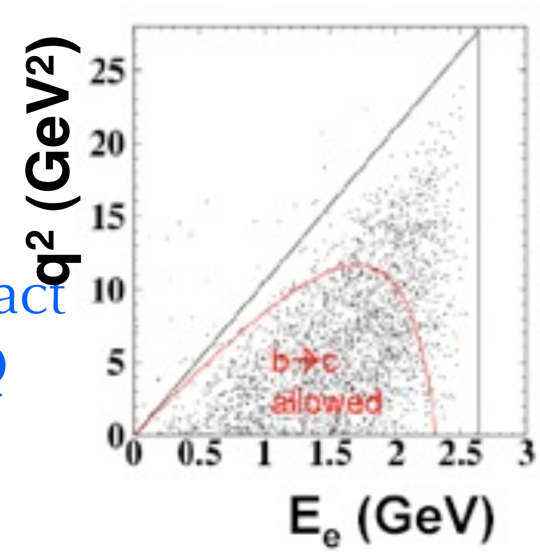
- Main uncertainty (±5%) from m_b^5 but we need a reasonable fraction of the rate to control theory uncertainty.

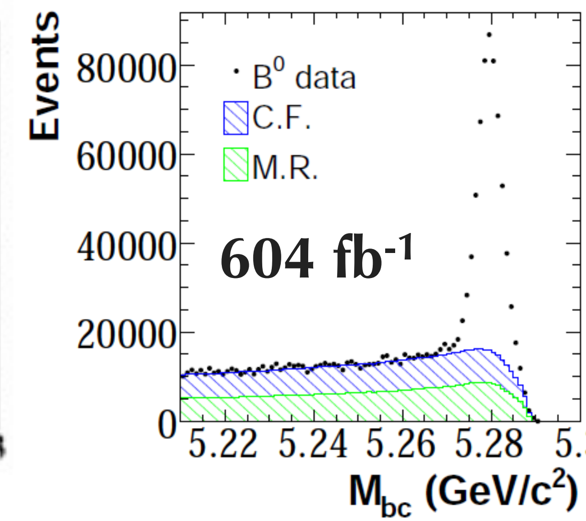
Multivariate analysis from Belle

Belle analysis exploits non-linear correlations between kinematic and event variables available in B-full recon sample to separate b→u and b→c.

PRL 104 2021801 (2010)

Boosted decision tree: use many event parameters from the full reconstruction sample: M_{miss}^2, impact parameters, Q_{total}, Q_{lepton}, N_{lepton}, Q (B), D* partial reco., N_{KS}, $N_{K\pm}$ ...





1.15x10⁶ Fully reconstructed *B*-mesons

Measure the partial BR, with p_{lepton}>1.0 GeV/c .

→**90 % total phase space!**

$$\Delta\mathcal{B} = \frac{N^{\Delta}_{b\to u}}{(2\epsilon^{\Delta}_{b\to u} N_{tag})}(1-\delta_{rad})$$

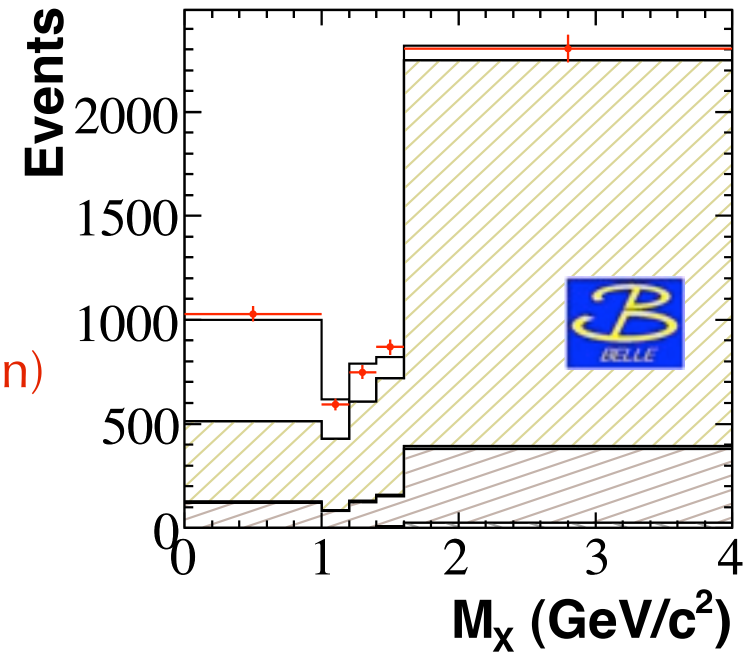
Source	# Events
BDT selected	5544 ± 54
scaled off-resonance	35 ± 18
wrong B_{tag}	825 ± 38
$X_u\ell\nu$	1032 ± 91
$X_c\ell\nu$	3615 ± 32
Secondary and fakes	38 ± 2

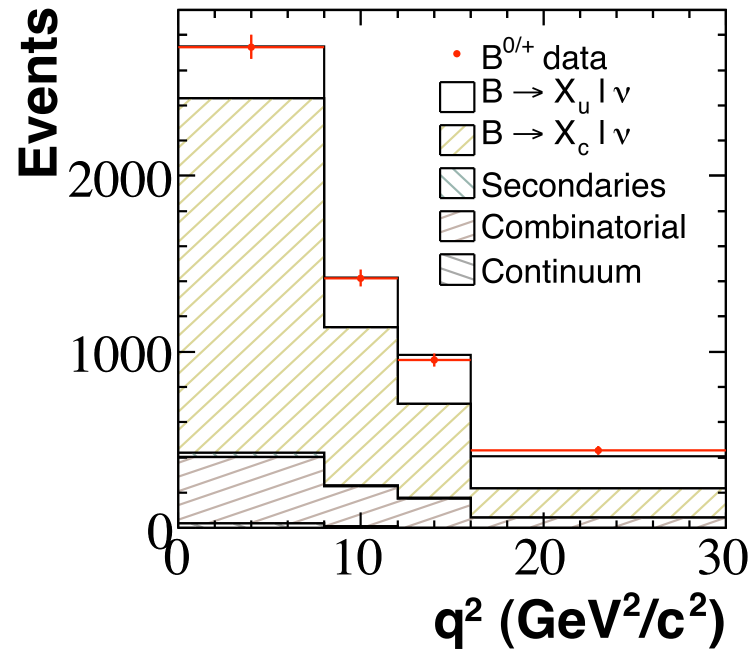
Inclusive $|V_{ub}|$ from Belle

PRL 104 2021801 (2010)

2D fit in q2 v M_X. (projections shown)

604 fb^{-1}





$$\Delta BR(p^*_{lep}>1.0\text{GeV}) = 1.963\ (1 \pm 0.088_{stat} \pm 0.081_{sys})\ 10^{-3}$$

Error breakdown in %

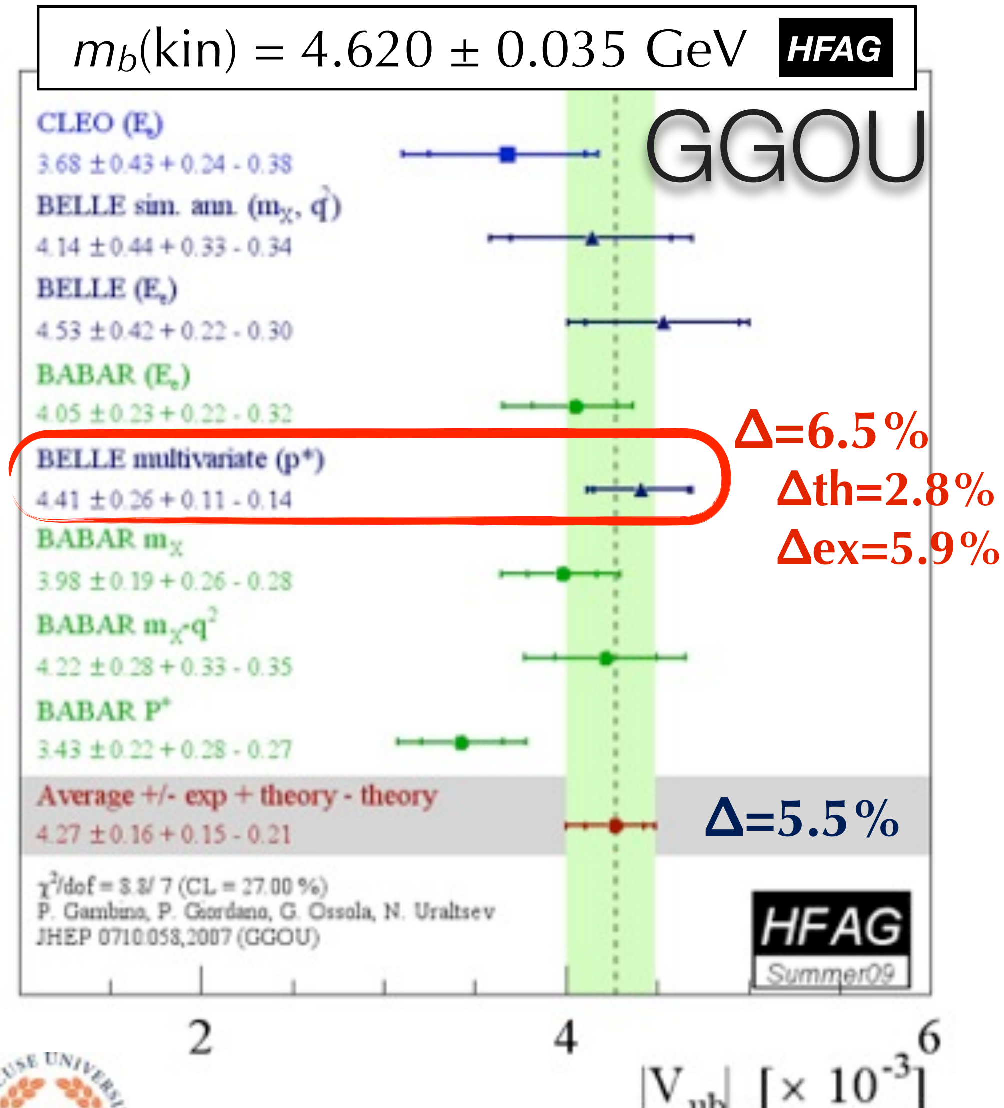
sys.	detector/other		B → Xu l ν			B → cl ν		
	Det.	M_{bc}	SF	Excl	ss	Inc.	FF	Ex. BR
8.1	**4.8**		**3.6**	**4.9**	**1.5**	**1.7**		

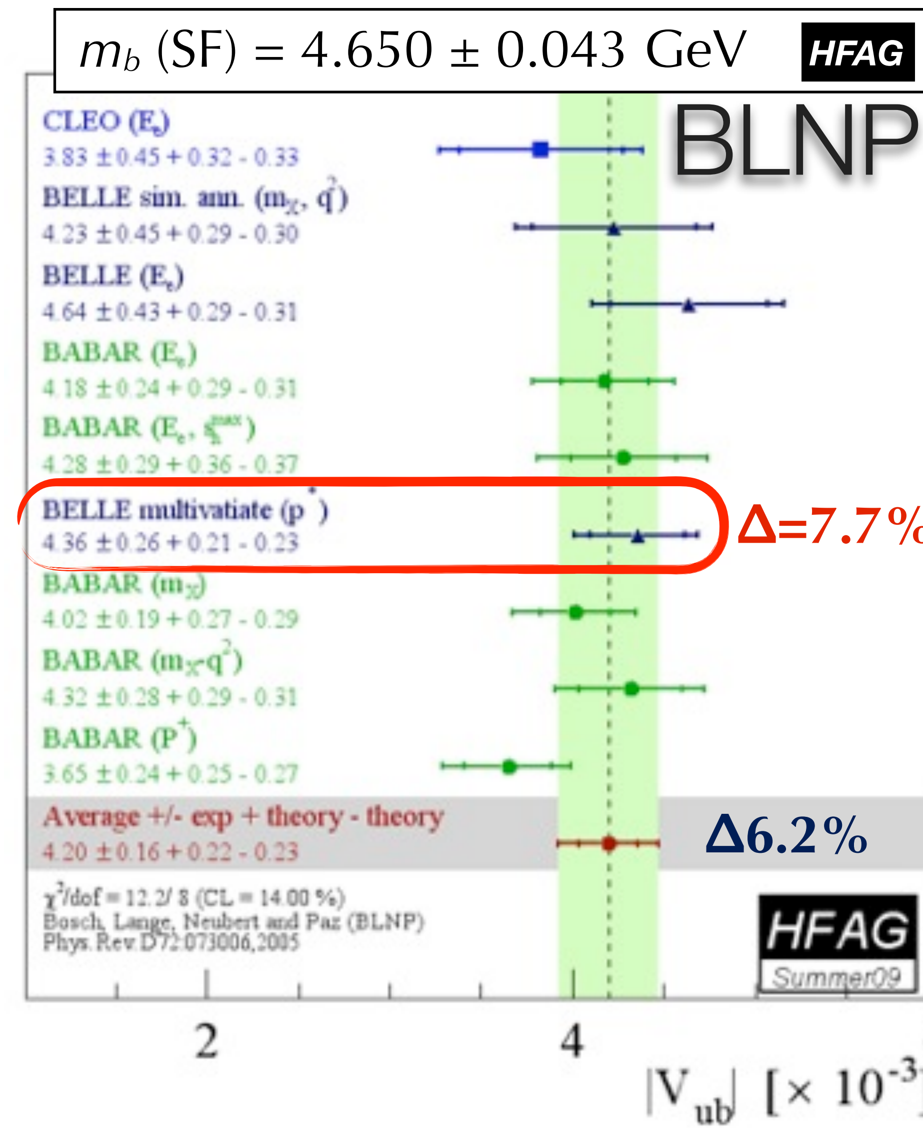
- **Gives single most precise |Vub|.**
- ***Lowest theory error*** ***on*** **|*Vub*|, owing to** ***greatest phase space coverage.***

Inclusive $|V_{ub}|$

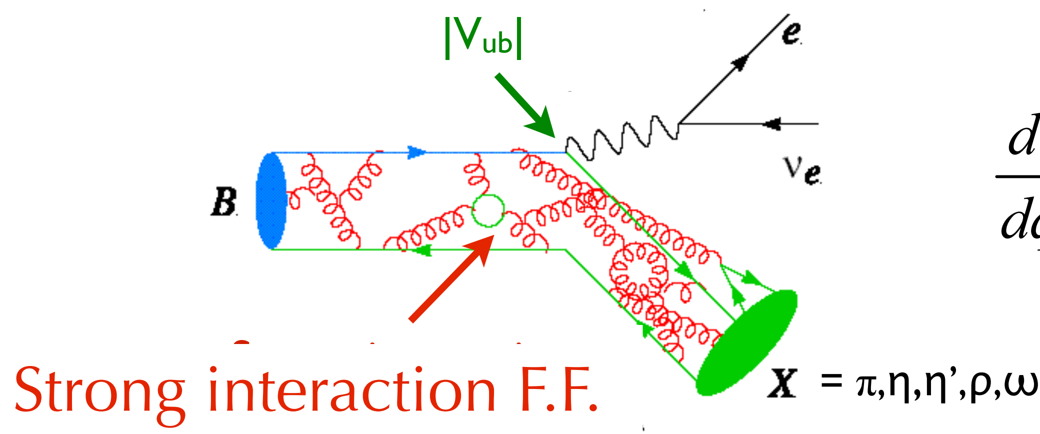
Extracted using several different methods and schemes e.g.

BLNP: PRD72:073006(2005)
DGE: JHEP 0601:097(2006)
GGOU: JHEP 0710:058(2007)
BLL: PRD64:113004(2001)





$|V_{ub}|$ from $B \to \pi\, l\, \nu$



$$\frac{d\Gamma}{dq^2}(B \to \pi l \nu) = \frac{G_F^2}{24\pi^3} p_\pi^3 \,|V_{ub}|^2 |f_+(q^2)|^2$$

Complementary experimental approaches:

- Untagged (with ν reconstruction)
- Semileptonic B tags
- Hadronic B tags

Independent samples, different systematic uncertainties

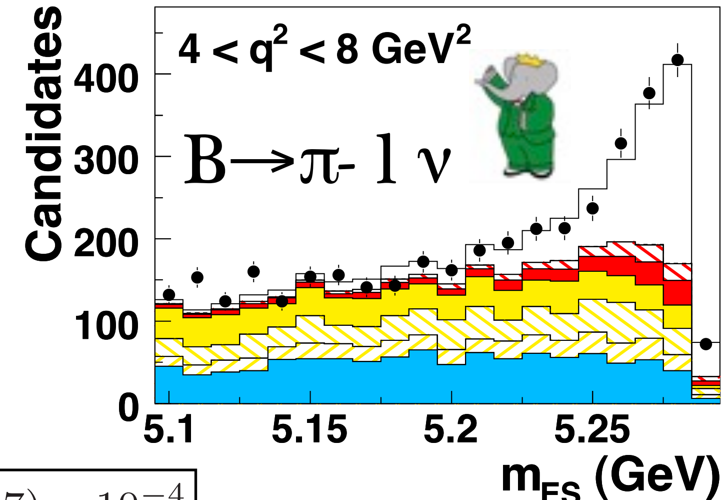
Form-factor calculations using different methods

- Unquenched lattice QCD (HPQCD, Fermilab)
- Light cone sum rules (Ball & Zwicky)
- Quark models (ISGW2)

Measurement in bins of q2→reduce model dependence

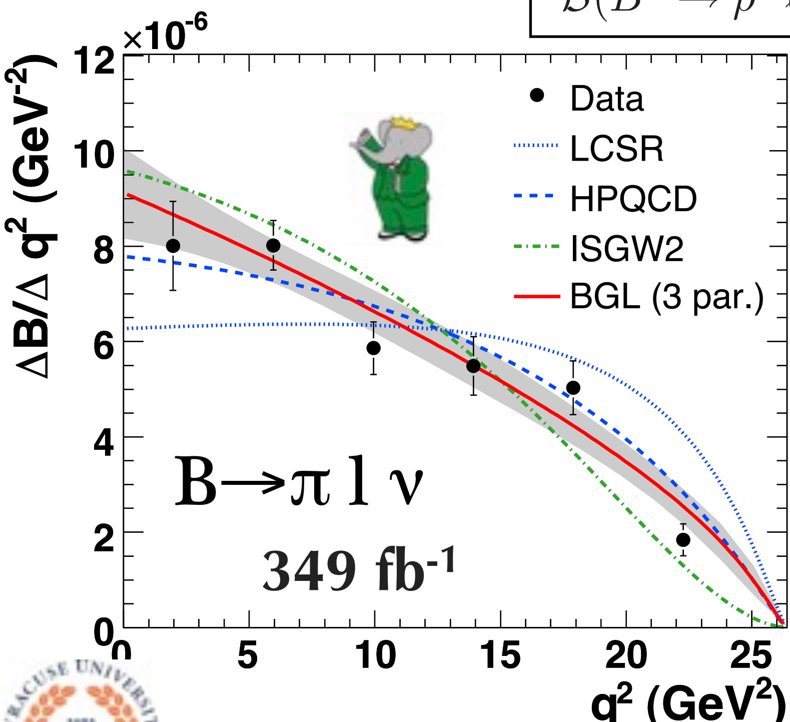
B→π/ρ l ν untagged from Babar

Latest preliminary untagged result from Babar measures simultaneously (π^-, π^0, ρ^-, ρ^0) imposing isospin.



Neural-Network selection,
Binned maximum likelihood fit to m_{ES} & ΔE in q^2 bins.

$$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$$
$$\mathcal{B}(B^0 \to \rho^- \ell^+ \nu) = (1.75 \pm 0.15 \pm 0.27) \times 10^{-4}$$



	q^2 Range (GeV^2)	$\Delta\zeta$ (ps^{-1})	$\lvert V_{ub}\rvert$ (10^{-3})
$B \to \pi\ell\nu$			
LCSR [15]	$0 - 16$	5.44±1.43	$3.63 \pm 0.12^{+0.59}_{-0.40}$
HPQCD [22]	$16 - 26.4$	2.02±0.55	$3.21 \pm 0.17^{+0.55}_{-0.36}$
LCSR [15]	$0 - 26.4$	7.72±2.32	$3.46 \pm 0.10^{+0.68}_{-0.43}$
HPQCD [22]	$0 - 26.4$	9.35±3.22	$3.14 \pm 0.09^{+0.68}_{-0.43}$
$B \to \rho\ell\nu$			
LCSR [16]	$0 - 16.0$	13.79	2.75 ± 0.24
LCSR [16]	$0 - 20.3$	17.15	2.58 ± 0.22
ISGW2 [14]	$0 - 20.3$	14.20	2.83 ± 0.24

Simultaneous Babar and Lattice fit

Model independent expression based on analyticity (z expansion) => full q2 range

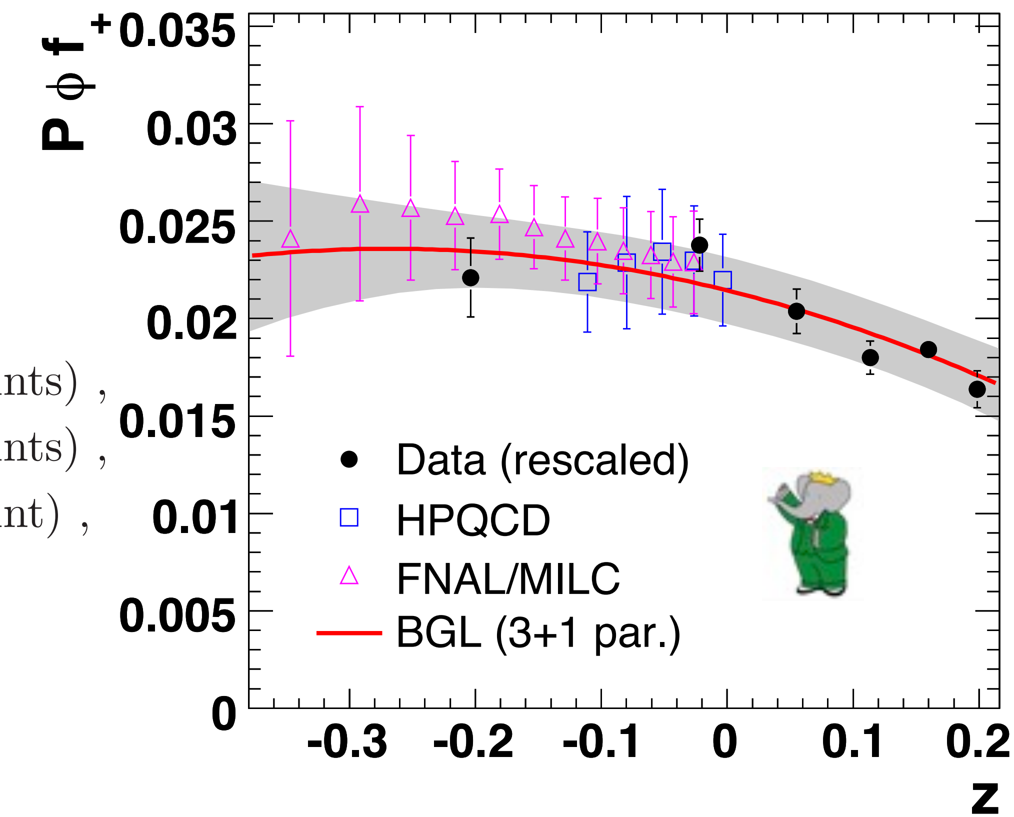
$$f_+(q^2) = \frac{1}{\mathcal{P}(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{k_{max}} a_k(q_0^2)[z(q^2, q_0^2)]^k \qquad z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}$$

$m_+ = M_B + m_\pi$ and q_0^2 is a free parameter

Simultaneous fit to data and lattice

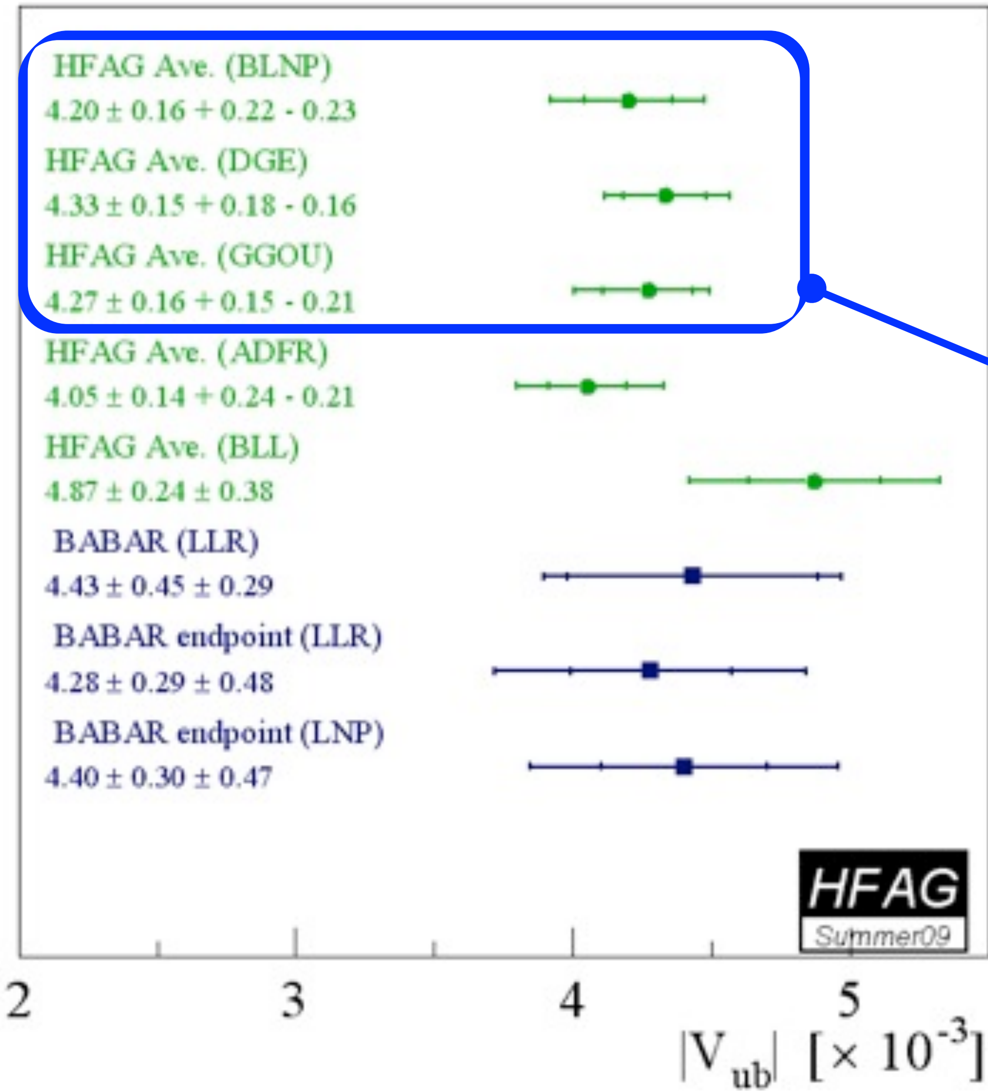
$$|V_{ub}| = (3.05 \pm 0.29) \times 10^{-3} \quad \text{FNAL/MILC (6 points)} ,$$
$$|V_{ub}| = (2.88 \pm 0.29) \times 10^{-3} \quad \text{FNAL/MILC (3 points)} ,$$
$$|V_{ub}| = (2.93 \pm 0.37) \times 10^{-3} \quad \text{FNAL/MILC (1 point)} ,$$
$$|V_{ub}| = (3.01 \pm 0.35) \times 10^{-3} \quad \text{HPQCD (1 point)} ,$$

precision @ 10%

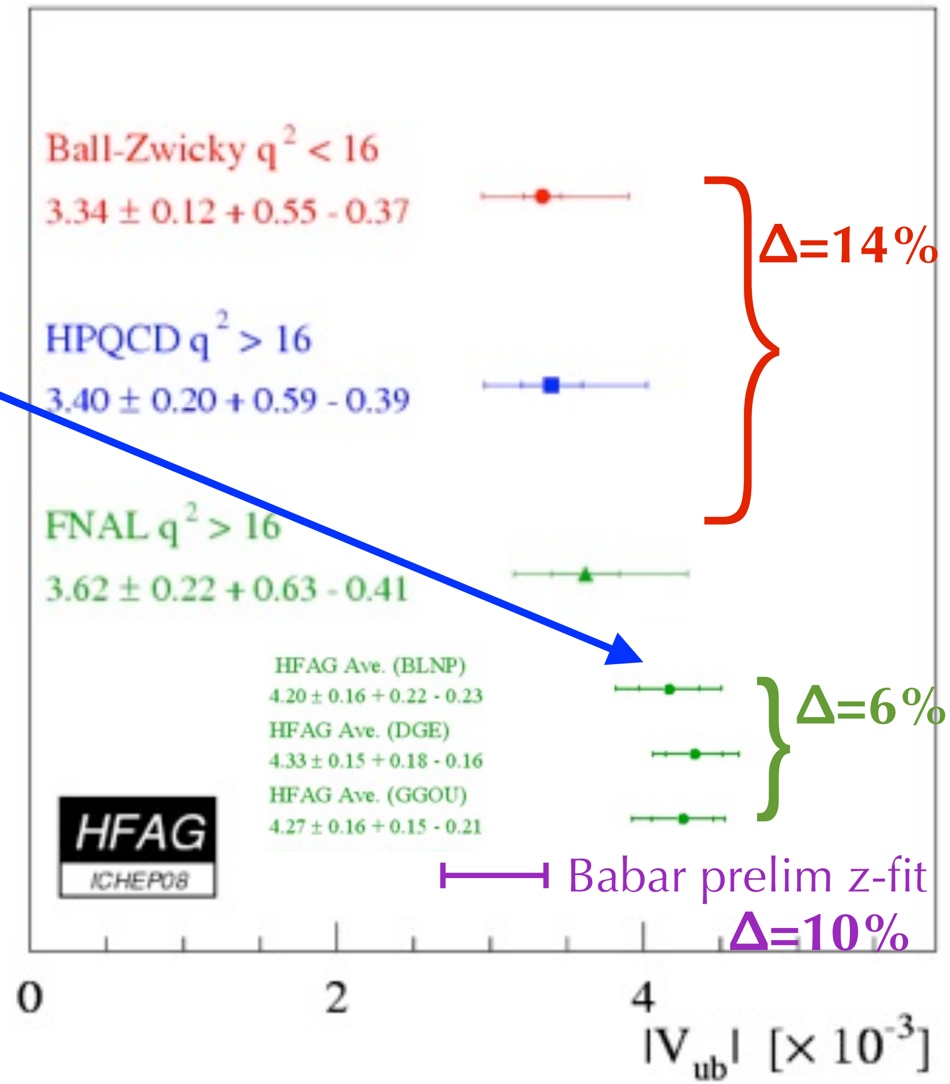


$|V_{ub}|$ summary Inclusive vs. Exclusive

Inclusive



Exclusive



Exclusive < Inclusive ~1-2σ, Greater discrepancy with *z*-fit.

Conclusions

Inclusive $|V_{cb}|$

High precision from HQE fits to moments (E_{lepton}, $E\gamma$, M_X and n_X).

Exclusive $|V_{cb}|$

Significant progress for $B \to Dl\nu$.

Important cross-checks $D \Leftrightarrow D^*$, $D^{*+} \Leftrightarrow D^{*0}$.

Inclusive $|V_{ub}|$

Limited by theory prediction of phase space acceptances.

New Belle result for 90% of phase space.

Exclusive $|V_{ub}|$, from $B \to \pi/\rho\ l\ \nu$

Limited by precision of form-factor calculations.

Combined fit to data and lattice points with reduced error.

2010 Precision

	$\|V_{cb}\|$	$\|V_{ub}\|$
inclusive	1-2%	6-7%
exclusive	3%	10%
difference	~2σ	~1-2σ