Measurements of the Unitarity Triangle Sides

$|V_{ub}|$ and $|V_{cb}|$ from Semileptonic B decays

Phillip Urquijo
Syracuse University
(for the Belle Collaboration)
Introduction

CKM matrix elements are fundamental parameters of the Standard Model and cannot be predicted.

Exploit the unitarity constraint to look for new physics → geometrical relation between CKM elements: angle from CP asymmetries, size from $|V_{\text{CKM}}|$.

Precision era: new physics may appear as a few percent disagreement: large new physics contributions to penguins would have already been seen.

We must make the green ring thinner → uncertainty dominated by $|V_{\text{ub}}|$.

Inclusive $|V_{\text{cb}}| : B \rightarrow X_c \ell \nu$

Exclusive $|V_{\text{cb}}| : B \rightarrow D^{(*)} \ell \nu$

Inclusive $|V_{\text{ub}}| : B \rightarrow X_u \ell \nu$

Exclusive $|V_{\text{ub}}| : B \rightarrow \pi \ell \nu$
Semileptonic B decays

tree level, short distance:

\[ b \rightarrow c e \nu \]

Decay properties depend directly on \(|V_{cb}| \& |V_{ub}|\) and \(m_b\) perturbative regime \((\alpha_s^n)\).
Semileptonic B decays

tree level, short distance:

\[ \text{Decay properties depend directly on } |V_{cb}| \text{ & } |V_{ub}| \text{ and } m_b \]

perturbative regime \((\alpha_s^n)\).

But quarks are bound by soft gluons: non-perturbative
long distance interactions of \(b\) quark with light quark.

+ long distance:
Exclusive Vs. Inclusive

One hadronic current.

**Inclusive decays** $b \rightarrow q \ell \nu$

- Weak quark decay + QCD corrections.
- $\Gamma_{\text{sl}}$ described by Heavy Quark Expansion in $(1/m_b)^n$ and $\alpha_s^k$

\[
\Gamma(B \rightarrow X_c \ell \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left[ 1 + A_{\text{ew}} \right] A_{\text{nonpert}} A_{\text{pert}}
\]

Non perturbative parameters need to be derived from data, i.e. from inclusive spectral moments of the semileptonic decay products.

**Theoretically easier, more precise.**

**Exclusive decays** $B \rightarrow X_q \ell \nu$

- Form factors: need lattice QCD.

\[
\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2}{24 \pi^2} |V_{ub}|^2 p_\pi^3 |f+(q^2)|^2
\]

i.e. Currently use $B \rightarrow \pi \ell \nu$ for $|V_{ub}|$ - one dominant form factor ($q^2$ shape and normalization needed).

*Experimentally clean, a check of inclusive methods.*

Phillip Urquijo, Moriond EW, March 2010
Inclusive decays: Big Picture

Semileptonic $B$ decay

Experimental Challenge:
Go from the measured shape → true shape:
shape in $B$ rest frame, QED corrections, detector resolution, accessible phase space, $X_c$lv model etc.
Moments from Babar

Recent moments results from Babar also include “mixed” moments PRD 81 032003 (2010)

Alternative extraction of the higher-order nonperturbative HQE parameters

\[ <n_x^2>^k: \quad n_x^2 = M_x^2 - 2 \Lambda E_x + \Lambda^2 \]

Different experiments in good agreement: confidence in OPE fits.

Phillip Urquijo, Moriond EW, March 2010
**$|V_{cb}|$ from Global Fit**

HFAG averages different measurements in the Kinetic and 1S schemes: 27 from Babar, 25 from Belle, 12 from CDF+CLEO+DELPHI.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>S.L. +Rad.</td>
<td>$</td>
<td>V_{cb}</td>
</tr>
<tr>
<td>$m_b^{\text{kin}}$ [GeV]</td>
<td>4.620±0.035</td>
<td>4.689±0.028</td>
</tr>
<tr>
<td>$\mu_\pi^2$ [GeV$^2$]</td>
<td>0.440±0.040</td>
<td>-0.336±0.022</td>
</tr>
</tbody>
</table>

Excellent agreement between 1S & kinetic scheme.

$\Delta |V_{cb}| / |V_{cb}| \sim 1\text{"}-2\%$ dominated by theory uncertainties.

arXiv:0808.1297

Phillip Urquijo, Moriond EW, March 2010
Exclusive decays: $B \to D(\ast)\ell \nu$

Differential decay rate:

$$\frac{d\Gamma}{dw}(B \to D\ell\nu) \sim (\text{Phase Space})|V_{cb}|^2 G(w)^2$$

$$\frac{d\Gamma}{dw}(B \to D^*\ell\nu) \sim (\text{Phase Space})|V_{cb}|^2 F(w)^2 \sum_{i=+,0,-} |H_i(w)|^2$$

Form factors can be parameterised:

$$G(w) = G(1)[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3], \quad z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w-1} + \sqrt{2}}$$

$$F(w) = ...$$

---

From experiment

$|V_{cb}| \times \text{F.F.} \ @ w=1 \ (0 \ recoil)$

$\rho_D, \rho_{D^*} \ (\text{F.F. slopes})$

---

From Lattice

$G(1)=1.074\pm0.024, \ NPPS \ 140, \ 461 \ (2005)$

$F(1)=0.921\pm0.024, \ PRD \ 79 \ 014506 \ (2009)$
B→D* l ν from Belle

• Study charged and neutral B decays:

• $B^0 \rightarrow D^* l^+ ν$, $D^* \rightarrow D^0 π^- \ arXiv:0810.1657$

• $B^± \rightarrow D^*0 l^+ ν$, $D^{*0} \rightarrow D^0π^0 \ arXiv:0910.3534$

• Measure $w$ and decay angles $θ_ℓ$, $θ_ν$, $χ$

• Fit 4-D decay rate $d^4Γ(B^+ → \bar{D}^*0 l^+ ν_ℓ) / dw d(cos θ_ℓ) d(cos θ_ν) dχ$

$B^0 → D^* l^+ ν \quad 140 \text{ fb}^{-1} \quad B^± → D^*0 l^+ ν$

Phillip Urquiijo, Moriond EW, March 2010
B→D* l ν from Belle

Results of 4-parameter HQET parameterization fit.

B+/B0 are consistent.

Relatively low values of F(1)|Vcb|.

Belle performs a model independent measurement of F.F. shapes.

Confirms use of Caprini et al. parameterisation.

<table>
<thead>
<tr>
<th></th>
<th>B^0→D* l^- ν</th>
<th>B^±→D*^0 l^- ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho^2 )</td>
<td>1.293±0.045±0.029</td>
<td>1.376±0.074±0.056</td>
</tr>
<tr>
<td>( R_1(1) )</td>
<td>1.495±0.050±0.062</td>
<td>1.620±0.091±0.092</td>
</tr>
<tr>
<td>( R_2(1) )</td>
<td>0.844±0.034±0.019</td>
<td>0.804±0.064±0.036</td>
</tr>
<tr>
<td>( BR(%) )</td>
<td>4.42±0.03±0.25</td>
<td>4.84±0.04±0.56</td>
</tr>
<tr>
<td>( F(1)</td>
<td>V_{cb}</td>
<td>)</td>
</tr>
<tr>
<td>( \chi^2/dof )</td>
<td>138.8/155</td>
<td>187.8/155</td>
</tr>
</tbody>
</table>

FIG. 11: Results of the fit of the helicity amplitudes (red crosses) compared to the prediction obtained by using the parametrization prescription by Caprini et al. [3] (solid black line). The left plot shows the results for \( \Gamma_{00} \), the right for \( \Gamma_T \). Only the statistical errors are shown.

\( \sum_{i} \gamma_i^0 + \sum_{i} \gamma_i^T \approx \chi^2/dof = 138.8/155 \) or a \( \chi^2 \) probability of \( P_{\chi^2} = 2.86\% \).

\( \sum_{i} \gamma_i^0 = 187.8/155 \) or a \( \chi^2 \) probability of \( P_{\chi^2} = 2.86\% \).

\( \gamma_i^0 \) and \( \gamma_i^T \) are the partial decay widths for each of the helicity components. Tables V and VI give the results of the fits, where the systematic errors quoted in these tables stem from the same sources as given in the breakdown in Table IV.
B→D(*)lν from Babar

Two recent, complementary, B→D l ν results from Babar.

- **Untagged**, simultaneous fit of B→D*l ν and B→Dl ν, PRD 79, 012002 (2009)
- **Hadronic B-tag** measurement, PRL 104 011802 (2010)

### Hadronic B-tag

| \(|V_{cb}| G(1) = (42.3 \pm 1.9 \pm 1.4) \times 10^{-3}\) |
| \(\rho_D^2=1.20\pm0.09\pm0.04\) |
| \(\text{BR}(B-\rightarrow D l ν)=(2.15\pm0.06\pm0.09)\%\) |

417 fb⁻¹
$|V_{cb}|$ from $B \rightarrow D(\ast)l \nu$

$|V_{cb}| G(1) = (42.3 \pm 0.7 \pm 1.3) \times 10^{-3}$

$|V_{cb}| F(1) = (35.75 \pm 0.42) \times 10^{-3}$

precision $\sim 4\%$

precision $\sim 3\%$, tension in ave.
$|V_{cb}|$ summary: Inclusive vs. Exclusive

Exclusive $|V_{cb}| \sim 2\sigma$ lower than inclusive

HFAG averages
Limiting factor in CKM precision tests; known much less well than $|V_{cb}|$
CKM suppressed $V_{ub} \sim 0.1 \times V_{cb}$ - therefore harder to measure.

The problem: $b \rightarrow c \ell \nu$ decay

\[ \frac{\Gamma(b \rightarrow u \ell \bar{\nu})}{\Gamma(b \rightarrow c \ell \bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50} \]
Inclusive $|V_{ub}|$ Measurement

• Cut away $b \rightarrow \text{clv}$: lose a part of the $b \rightarrow \text{ulv}$ signal.

• We measure

\[ \Gamma(B \rightarrow X_u \ell \nu) \times f_C = |V_{ub}|^2 \zeta_C \]

Cut-dependent constant predicted by theory

Total $b \rightarrow \text{ulv}$ rate

Fraction of the signal that pass the cut

→ corrected for QCD, motion of $b$-quark

\[ \Gamma(B \rightarrow X_u \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left[ 1 - O \left( \frac{\alpha_s}{\pi} \right) - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \cdots \right] \]

• Main uncertainty ($\pm 5\%$) from $m_b^5$ but we need a reasonable fraction of the rate to control theory uncertainty.

$ f_C \sim 25\%$ for $E_\ell > 2.0$ GeV,

$ f_C \sim 38\%$ for $q^2 > 8$ GeV$^2$,

$ f_C \sim 65\%$ for $M_x < 1.7$ GeV
Multivariate analysis from Belle

Belle analysis exploits non-linear correlations between kinematic and event variables available in B-full recon sample to separate $b\rightarrow u$ and $b\rightarrow c$.

**PRL 104 2021801 (2010)**

Boosted decision tree: use many event parameters from the full reconstruction sample: $M_{\text{miss}}^2$, impact parameters, $Q_{\text{total}}$, $Q_{\text{lepton}}$, $N_{\text{lepton}}$, $Q_{(B)}$, $D^*$ partial reco., $N_{\text{KS}}$, $N_{K\pm}$ ...

Measure the partial BR, with $p_{\text{lepton}} > 1.0$ GeV/c .

→ 90% total phase space!

$$\Delta B = \frac{N_{b\rightarrow u}^\Delta}{(2\epsilon_{b\rightarrow u}^\Delta N_{\text{tag}})} (1 - \delta_{\text{rad}})$$

1.15x10^6 Fully reconstructed $B$-mesons

<table>
<thead>
<tr>
<th>Source</th>
<th># Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>BDT selected</td>
<td>5544 ± 54</td>
</tr>
<tr>
<td>scaled off-resonance</td>
<td>35 ± 18</td>
</tr>
<tr>
<td>wrong $B_{\text{tag}}$</td>
<td>825 ± 38</td>
</tr>
<tr>
<td>$X_u \ell\nu$</td>
<td>1032 ± 91</td>
</tr>
<tr>
<td>$X_c \ell\nu$</td>
<td>3615 ± 32</td>
</tr>
<tr>
<td>Secondary and fakes</td>
<td>38 ± 2</td>
</tr>
</tbody>
</table>

Phillip Urquijo, Moriond EW, March 2010
Inclusive $|V_{ub}|$ from Belle

PRL 104 2021801 (2010)

2D fit in q$^2$ v $M_x$. (projections shown)

604 fb$^{-1}$

$\Delta BR(p^*_{lep}>1.0\text{GeV}) = 1.963 \ (1 \pm 0.088_{\text{stat}} \pm 0.081_{\text{sys}}) \times 10^{-3}$

**Error breakdown in %**

<table>
<thead>
<tr>
<th>sys.</th>
<th>detector/other</th>
<th>$B \rightarrow X_u \ell \nu$</th>
<th>$B \rightarrow c\ell \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Det.</td>
<td>$M_{bc}$</td>
<td>SF</td>
</tr>
<tr>
<td>8.1</td>
<td>4.8</td>
<td>3.6</td>
<td>4.9</td>
</tr>
</tbody>
</table>

• Gives single most precise $|V_{ub}|$.
• Lowest theory error on $|V_{ub}|$, owing to greatest phase space coverage.
Inclusive $|V_{ub}|$

Extracted using several different methods and schemes e.g.

$mb(\text{kin}) = 4.620 \pm 0.035 \text{ GeV}$  

$mb\ (\text{SF}) = 4.650 \pm 0.043 \text{ GeV}$

$\Delta = 6.5\%$  

$\Delta_{\text{th}} = 2.8\%$  

$\Delta_{\text{ex}} = 5.9\%$

BLL: PRD64:113004(2001)
$|V_{ub}|$ from $B \rightarrow \pi l \nu$

\[ \frac{d\Gamma}{dq^2}(B \rightarrow \pi l \nu) = \frac{G_F^2}{24\pi^3} p_\pi^3 |V_{ub}|^2 |f_+(q^2)|^2 \]

**Strong interaction F.F.**

$X = \pi, \eta, \eta', \rho, \omega$

**Complementary experimental approaches:**

- Untagged (with $\nu$ reconstruction)
- Semileptonic B tags
- Hadronic B tags

**Form-factor calculations using different methods**

- Unquenched lattice QCD (HPQCD, Fermilab)
- Light cone sum rules (Ball & Zwicky)
- Quark models (ISGW2)

**Independent samples, different systematic uncertainties**

**Measurement in bins of $q^2$ → reduce model dependence**
B→π/0 l ν untagged from Babar

Latest preliminary untagged result from Babar measures simultaneously ($\pi^-, \pi^0, \rho^-, \rho^0$) imposing isospin.

Neural-Network selection,
Binned maximum likelihood fit to $m_{ES}$ & $\Delta E$ in $q^2$ bins.

$$B(B^0 \rightarrow \pi^{-} \ell^{+} \nu) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$$

$$B(B^0 \rightarrow \rho^{-} \ell^{+} \nu) = (1.75 \pm 0.15 \pm 0.27) \times 10^{-4}$$
Simultaneous Babar and Lattice fit

Model independent expression based on analyticity (z expansion) => full q^2 range

\[ f_+(q^2) = \frac{1}{\mathcal{P}(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{k_{max}} a_k(q_0^2)[z(q^2, q_0^2)]^k \]

\[ z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}} \]

\[ m_+ = M_B + m_\pi \text{ and } q_0^2 \text{ is a free parameter} \]

Simultaneous fit to data and lattice

\[
\begin{align*}
|V_{ub}| &= (3.05 \pm 0.29) \times 10^{-3} & \text{FNAL/MILC (6 points)}, \\
|V_{ub}| &= (2.88 \pm 0.29) \times 10^{-3} & \text{FNAL/MILC (3 points)}, \\
|V_{ub}| &= (2.93 \pm 0.37) \times 10^{-3} & \text{FNAL/MILC (1 point)}, \\
|V_{ub}| &= (3.01 \pm 0.35) \times 10^{-3} & \text{HPQCD (1 point)},
\end{align*}
\]

precision @ 10%
$|V_{ub}|$ summary Inclusive vs. Exclusive

**Inclusive**

- HFAG Ave. (BLNP)
  - $4.20 \pm 0.16 + 0.22 - 0.23$
- HFAG Ave. (DGE)
  - $4.33 \pm 0.15 + 0.18 - 0.16$
- HFAG Ave. (GGOU)
  - $4.27 \pm 0.16 + 0.15 - 0.21$
- HFAG Ave. (ADFR)
  - $4.05 \pm 0.14 + 0.24 - 0.21$
- HFAG Ave. (BLL)
  - $4.87 \pm 0.24 + 0.38$
- BABAR (LLB)
  - $4.43 \pm 0.45 + 0.29$
- BABAR endpoint (LLB)
  - $4.28 \pm 0.29 + 0.48$
- BABAR endpoint (LNP)
  - $4.40 \pm 0.30 + 0.47$

**Exclusive**

- Ball-Zwicky $q^2 < 16$
  - $3.34 \pm 0.12 + 0.55 - 0.37$
- HPQCD $q^2 > 16$
  - $3.40 \pm 0.20 + 0.59 - 0.39$
- FNAL $q^2 > 16$
  - $3.62 \pm 0.22 + 0.63 - 0.41$

Exclusive $<$ Inclusive $\sim 1-2\sigma$, Greater discrepancy with z-fit.
Conclusions

Inclusive $|V_{cb}|$

High precision from HQE fits to moments ($E_{\text{lepton}}, E_{\gamma}, M_X$ and $n_X$).

Exclusive $|V_{cb}|$

Significant progress for $B \rightarrow D l \nu$.

Important cross-checks $D \leftrightarrow D^*$, $D^{*+} \leftrightarrow D^{*0}$.

Inclusive $|V_{ub}|$

Limited by theory prediction of phase space acceptances.

New Belle result for 90% of phase space.

Exclusive $|V_{ub}|$, from $B \rightarrow \pi/\rho l \nu$

Limited by precision of form-factor calculations.

Combined fit to data and lattice points with reduced error.

| 2010 Precision  | $|V_{cb}|$  | $|V_{ub}|$  |
|-----------------|-----------|-----------|
| inclusive       | 1-2%      | 6-7%      |
| exclusive       | 3%        | 10%       |
| difference      | $\sim 2\sigma$ | $\sim 1-2\sigma$ |