Measurement of $\sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0)$ via ISR Using 527 fb$^{-1}$ of Belle Data

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1. Motivation
2. Method
3. Cross Section Determination
4. $\pi\pi$ Mass Distributions
Multi-hadron cross sections determine HVP contributions for muon (g-2) and running of $\alpha_{\text{QED}}$

- Muon (g-2) hints at new physics beyond the Standard Model
  - New experiment aims at 4-fold increased precision
  - Improved cross sections required
- $\alpha_{\text{QED}}$ at Z mass important in global EW fits
  - Requires improved cross sections as well

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Low energy HVP contribution is from measured $R(s)$ ratio

$$a_{\mu}^{HVP} = \left( \frac{\alpha m_{\mu}}{3\pi} \right)^2 \int_{4m_{\pi}^2}^{\infty} \frac{ds}{s^2} R(s) \hat{K}(s)$$

- $\sqrt{s} = $ c.m. frame energy.
- Amplifies low invariant mass $R(s)$.
- Obtained from the optical theorem.

$$R(s) \equiv \sigma(e^+e^- \rightarrow \text{hadrons}) \left( \frac{4\pi\alpha^2}{3s} \right)$$

$$\hat{K}(s) \equiv \frac{3s}{m_{\mu}^2} K(s)$$

$$K(s) \equiv \int_0^1 dx \frac{x^2(1-x)}{x^2 + (s/m_{\mu}^2)(1-x)}$$

Relative importance of exclusive cross sections, ordered by impact on muon \((g-2)\) ... \(\alpha_{\text{QED}}\) slightly different

<table>
<thead>
<tr>
<th>e⁺e⁻ process</th>
<th>Contribution to (a_{\mu}^{\text{had, LO}}) ([x10^{-10}])</th>
<th>Total error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>val ± sta. ± process sys. ± common sys.</td>
<td></td>
</tr>
<tr>
<td>(\pi^+\pi^-)</td>
<td>507.80 ± 1.22 ± 2.50 ± 0.56</td>
<td>2.88</td>
</tr>
<tr>
<td>(\pi^+\pi^-\pi^0)</td>
<td>46.00 ± 0.42 ± 1.03 ± 0.98</td>
<td>1.48</td>
</tr>
<tr>
<td>(K^+K^-)</td>
<td>21.63 ± 0.27 ± 0.58 ± 0.36</td>
<td>0.73</td>
</tr>
<tr>
<td>(\pi^+\pi^-\pi^0\pi^0)</td>
<td>18.01 ± 0.14 ± 0.17 ± 0.40</td>
<td>0.46</td>
</tr>
<tr>
<td>(\pi^+\pi^+\pi^-)</td>
<td>13.35 ± 0.10 ± 0.43 ± 0.29</td>
<td>0.53</td>
</tr>
<tr>
<td>(K_S^0K_L^0)</td>
<td>12.96 ± 0.18 ± 0.25 ± 0.24</td>
<td>0.39</td>
</tr>
<tr>
<td>(\pi^0\gamma)</td>
<td>4.42 ± 0.08 ± 0.13 ± 0.12</td>
<td>0.19</td>
</tr>
<tr>
<td>(K\bar{K}\pi) (partly from isospin)</td>
<td>2.39 ± 0.07 ± 0.12 ± 0.08</td>
<td>0.16</td>
</tr>
<tr>
<td>(K\bar{K}\pi\pi) (partly from isospin)</td>
<td>1.35 ± 0.09 ± 0.38 ± 0.03</td>
<td>0.39</td>
</tr>
<tr>
<td>(\pi^+\pi^-\eta)</td>
<td>1.15 ± 0.06 ± 0.08 ± 0.03</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Total</strong> (a_{\mu}^{\text{had, LO}})</td>
<td><strong>692.3 ± 1.4 ± 3.1 ± 2.4 ± 0.2_{\psi} ± 0.3_{\text{QCD}}</strong></td>
<td><strong>4.18</strong></td>
</tr>
</tbody>
</table>

\[\pi^+\pi^-\pi^0 \rightarrow 6.6\% \text{ of Total } a_{\mu}^{\text{had, LO}}\]

Long history measuring $\sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0)$

$e^+e^-\rightarrow\pi^+\pi^-\pi^0$ Cross Section

$\sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0)$ [nb]
Long history measuring $\sigma(e^+e^{-}\rightarrow\pi^+\pi^0\pi^0)$

$e^+e^-\rightarrow\pi^+\pi^-\pi^0$ Cross Section

$\sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0)$ [nb]

$\sqrt{s}$ [GeV]
Belle $\sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0)$ measurement covers the full range.
Fixed-energy experiments can measure a full range of center-of-mass energies (below c.m. frame energy of the experiment) via ISR.

Initial State Radiation:
\( \gamma \) leaves \( e \) virtual.
\( \gamma \) lowers \( e^+e^- \) invariant mass.

Final State Radiation:
Suppressed by kinematic cuts to < 1%

Goal: Determine absolute cross section from threshold to above the J/Ψ. No one has reported that from a single experiment

\[ \sigma_i = \frac{N_i^{corr}}{L_i \epsilon_i^{corr} B_{\pi^0 \rightarrow \gamma\gamma}} \]

- \( \sigma_i \) = visible cross section for \( i^{th} \) bin
- \( B_{\pi^0 \rightarrow \gamma\gamma} \) = branching ratio (PDG)

Note: in a later step, we will apply radiative corrections to produce the Born cross section
Goal: Determine absolute cross section from threshold to above the J/Ψ. No one has reported that from a single experiment

\[
\sigma_i = \frac{N_i^{corr}}{L_i \epsilon_i^{corr} B_{\pi^0 \rightarrow \gamma \gamma}}
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- \(\sigma_i\) = visible cross section for \(i^{th}\) bin
- \(B_{\pi^0 \rightarrow \gamma \gamma}\) = branching ratio (PDG)
- \(N_i^{corr}\) = corrected signal yield (background subtractions and mass unfolding)

Note: in a later step, we will apply radiative corrections to produce the Born cross section
Goal: Determine absolute cross section from threshold to above the $J/\Psi$. No one has reported that from a single experiment

$$\sigma_i = \frac{N_{i}^{corr}}{L_i \epsilon_i^{corr} B_{\pi^0 \rightarrow \gamma \gamma}}$$

- $\sigma_i = \textit{visible}$ cross section for $i^{th}$ bin
- $B_{\pi^0 \rightarrow \gamma \gamma} = \text{branching ratio (PDG)}$
- $N_{i}^{corr} = \text{corrected signal yield (background subtractions and mass unfolding)}$
- $L_i = \text{effective integrated luminosity (includes radiator function for ISR photon)}$

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\[ \sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B_{\pi^0 \rightarrow \gamma\gamma}} \]

- \( \sigma_i = \) visible cross section for \( i^{th} \) bin
- \( B_{\pi^0 \rightarrow \gamma\gamma} = \) branching ratio (PDG)
- \( N_i^{corr} = \) corrected signal yield (background subtractions and mass unfolding)
- \( L_i = \) effective integrated luminosity (includes radiator function for ISR photon)
- \( \varepsilon_i = \) corrected detector efficiency (realistic event generator with NLO ISR; MC corrections using Data)

Note: in a later step, we will apply radiative corrections to produce the Born cross section
Data and Monte Carlo flow through the same analysis chain

\[ \sigma_i = \frac{N_i^{corr}}{L \cdot \epsilon_i^{corr} \cdot B_{\pi^0 \rightarrow \gamma\gamma}} \]
Data and Monte Carlo flow through the same analysis chain

Data $\rightarrow N_i^{corr}$

526.6 fb$^{-1}$

Physics Skim

Simulation $\rightarrow \varepsilon_i^{corr}$

Phokhara 6.2

GEANT

NLO ISR; FSR; Form Factors

Physics Skim; Trigger Simulation

Goal: Detector eff

Pre Kinematic Fit Cuts

Mostly topological and quality of event

$$\sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B_{\pi^0 \rightarrow \gamma\gamma}}$$
Data and Monte Carlo flow through the same analysis chain

Data $\rightarrow N_i^{corr}$

Simulation $\rightarrow \varepsilon_i^{corr}$

- **Physics Skim**
  - **Phokhara 6.2**
  - NLO ISR; FSR; Form Factors
  - Physics Skim; Trigger Simulation
  - Goal: Detector eff

- **GEANT**
  - Mostly topological and quality of event

- **Pre Kinematic Fit Cuts**
  - Good kinematic fit of $\pi^+\pi^-\gamma\gamma$; PID applied here

- **Post - Kinematic Fit Cuts**

**Equation**

$$\sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B_{\pi^0 \rightarrow \gamma\gamma}}$$
Data and Monte Carlo flow through the same analysis chain

\[ \text{Data} \rightarrow N_i^{corr} \]

\[ \text{Simulation} \rightarrow \varepsilon_i^{corr} \]

\[ 526.6 \, \text{fb}^{-1} \]

\[ \text{Physics Skim} \]

\[ \text{Phokhara 6.2} \]

\[ \text{GEANT} \]

\[ \text{NLO ISR; FSR; Form Factors} \]

\[ \text{Physics Skim; Trigger Simulation} \]

\[ \text{Goal: Detector eff} \]

\[ \sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B_{\pi^0 \rightarrow \gamma\gamma}} \]

Mostly topological and quality of event

Good kinematic fit of \( \pi^+\pi^-\gamma\gamma \); PID applied here

\( m(\gamma\gamma) \) distribution built to search for \( \pi^0 \).
Data and Monte Carlo flow through the same analysis chain

**Data** → \( N_i^{corr} \)

**Simulation** → \( \varepsilon_i^{corr} \)

- **Physics Skim**
  - 526.6 fb\(^{-1}\)

- **GEANT**
  - NLO ISR; FSR; Form Factors
  - Physics Skim; Trigger Simulation
  - Goal: Detector eff

- **Data**

- **Monte Carlo**

- **Pre Kinematic Fit Cuts**
  - Mostly topological and quality of event

- **Post - Kinematic Fit Cuts**
  - Good kinematic fit of \( \pi^+\pi^-\gamma\gamma\gamma \); PID applied here

- **Background subtraction**
  - \( m(\gamma\gamma) \) distribution built to search for \( \pi^0 \).

- **Mass Unfolding**

\[ \sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B_{\pi^0 \rightarrow \gamma\gamma}} \]
Comparison of MC and Data: 1

**ISR Photon Energy**

Kinematic fit 4-momentum constraint leads to a sharp $E(\gamma_{ISR})$ distribution

**ISR Photon Angle**

Data kfit (no bkg sub)
- Data pre-kfit (no bkg sub)
- MC kfit
- MC pre-kfit
- MC phok (greater cm energy $\gamma_{ISR}$)
Comparison of MC and Data: 2

Charged Pion Momentum

Charged Pion Angle

- Data kfit (no bkg sub)
- Data pre-kfit (no bkg sub)
- MC kfit
- MC pre-kfit
- MC phok (greater cm energy $\gamma_{IS}$)
Neutral pion mass distributions are created for each $\pi^+\pi^-\pi^0$ mass bin and **sideband** subtraction is used to determine the signal yield.

Very little background in the sidebands. A 3-bin running average is used.
Background yield in 5 MeV bins across full energy range
Detector efficiency is calculated with a quadratic polynomial fitted to MC detector efficiency

<table>
<thead>
<tr>
<th>Efficiency Correction</th>
<th>R ()</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID Efficiency</td>
<td>0.928±0.018</td>
</tr>
</tbody>
</table>

\[ \chi^2 / \text{ndf} = 721.3 / 580 \]

- \( p_0 = 0.01895 \pm 0.00073 \)
- \( p_1 = 0.05043 \pm 0.00115 \)
- \( p_2 = -0.008061 \pm 0.000374 \)

Pre-corrected Efficiency Averaged

MC in 5 MeV bins
Signal mass spectrum is unfolded to remove detector smearing. Important at resonances
Signal mass spectrum is unfolded to remove detector smearing. Important at resonances.
Final* Belle $\sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0)$
(selected mass regions; linear scale)

*Systematic errors, background leakage, and small radiative correction checks to be completed in near future
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(selected mass regions; linear scale)

*Systematic errors, background leakage, and small radiative correction checks to be completed in near future
$m(\pi^+\pi^-)$ distributions

$\omega$-region

$\phi$-region

$\omega'$-$\omega''$-region

PHOKHARA model needs correcting; small effect on total cross section.
m(\(\pi^+\pi^0\)) distributions

**ω-region**

**φ-region**

**ω'-ω''-region**

**J/ψ-region**

PHOKHARA model needs correcting; small effect on total cross section.
BaBar and BESIII obtain similar $2\pi$ Dalitz plots

BaBar $J/\psi \rightarrow \pi^+\pi^-\pi^0$

BESIII preliminary


## Preliminary Systematic Uncertainties

<table>
<thead>
<tr>
<th>Systematic Uncertainty</th>
<th>$\sigma(e^+e^-\rightarrow\pi^+\pi^0\pi^0)$ relative error (%)</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID Efficiency</td>
<td>2.8*</td>
<td>Efficiency Correction From Data</td>
</tr>
<tr>
<td>Track Finding Efficiency</td>
<td>0.7</td>
<td>From Data</td>
</tr>
<tr>
<td>Total Integrated Luminosity</td>
<td>1.4</td>
<td>MC Bhabha Generator Accuracy</td>
</tr>
<tr>
<td>Trigger Efficiency</td>
<td>Work In Progress</td>
<td>Variation Of Trigger Masks</td>
</tr>
<tr>
<td>$E(\gamma)$ Cuts</td>
<td>1.2</td>
<td>Variation Of Cuts</td>
</tr>
<tr>
<td>$\theta(\gamma_{\text{ISR}})$ Cut</td>
<td>3.8*</td>
<td>Variation Of Cuts</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>2.8*</td>
<td>Examine Largest Background</td>
</tr>
<tr>
<td>PID Cuts</td>
<td>0.94</td>
<td>Variation Of Cuts</td>
</tr>
<tr>
<td>Pre-Kfit $p^\mu$ Cuts</td>
<td>0.58</td>
<td>Variation Of Cuts</td>
</tr>
<tr>
<td>$m(\pi^0)$ Signal-Range</td>
<td>0.76</td>
<td>Variation Of Cuts</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$&gt; 5.9$</td>
<td>Add In Quadrature</td>
</tr>
</tbody>
</table>

*Goal: Total systematic at the ~5% level*

* Conservative estimate: Error can be lowered with more work*
Conclusions

$\sigma(e^+e^-\to \pi^+\pi^-\pi^0)$ measured using 527 fb$^{-1}$ from threshold to 3.5 GeV (from a single experiment)

- $\omega'$-$\omega''$-region agrees with BaBar, not DM2
Conclusions

\[ e^+e^- \rightarrow \pi^+\pi^-\pi^0 \] Cross Section

- \( \sigma(e^+e^-\rightarrow\pi^+\pi^-\pi^0) \) measured using 527 fb\(^{-1}\) from threshold to 3.5 GeV (from a single experiment)
- \( \omega'-\omega''\)-region agrees with BaBar, not DM2
- Work to be done
  - Final systematic errors
  - Include radiative corrections using PHOKHARA and PHOTOS
  - Include \( \sigma(e^+e^-\rightarrow\pi^+\pi^-\eta) \), \( \sigma(e^+e^-\rightarrowK^+K^-\pi^0) \), and \( \sigma(e^+e^-\rightarrowK^+K^-\eta) \) measurement (similar topology)
Backup Slides
The international Belle experiment operates at a B-Factory, and uses a general purpose detector.
Relative importance of inclusive cross section energy ranges on muon \((g-2)\) and \(\alpha_{\text{QED}}\)

The analysis uses exp31 to exp63; Y(4S) data

\[ L_{\text{eff}} \text{ [nb}^{-1}] \text{ for 5 MeV binning} \]

\[ L_i = \text{effective integrated luminosity in the } i\text{th } m \text{ bin} \]

\[ \frac{dL}{dm} = \frac{\alpha}{\pi x} \left[ (2 - 2x + x^2) \log \frac{1+C}{1-C} - x^2 C \right] \frac{2m}{s} L \]

\[ \frac{dL}{dm} = \text{ISR differential luminosity} \]

\[ x = 1 - \frac{m^2}{s} \quad C = \cos(\theta_0) \]

\[ \theta_0 \leq \theta_{y,\text{ISR}} \leq 180^\circ - \theta_0 \]

<table>
<thead>
<tr>
<th>exp#</th>
<th>L (pb(^{-1}))</th>
<th>(\Delta L^2) (pb(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>17901.92</td>
<td>8.9822</td>
</tr>
<tr>
<td>33</td>
<td>17756.00</td>
<td>8.7539</td>
</tr>
<tr>
<td>35</td>
<td>16837.48</td>
<td>8.2018</td>
</tr>
<tr>
<td>37a</td>
<td>28128.55</td>
<td>13.8404</td>
</tr>
<tr>
<td>37b</td>
<td>33464.76</td>
<td>16.6381</td>
</tr>
<tr>
<td>39</td>
<td>41189.63</td>
<td>20.4111</td>
</tr>
<tr>
<td>41a</td>
<td>31259.02</td>
<td>15.5116</td>
</tr>
<tr>
<td>41b</td>
<td>27489.32</td>
<td>13.8025</td>
</tr>
<tr>
<td>43</td>
<td>56028.39</td>
<td>28.0298</td>
</tr>
<tr>
<td>45a</td>
<td>4569.25</td>
<td>37.9709</td>
</tr>
<tr>
<td>45b</td>
<td>8399.37</td>
<td>69.4475</td>
</tr>
<tr>
<td>47</td>
<td>37338.25</td>
<td>310.0339</td>
</tr>
<tr>
<td>49</td>
<td>27330.11</td>
<td>225.7491</td>
</tr>
<tr>
<td>51</td>
<td>39850.56</td>
<td>328.5978</td>
</tr>
<tr>
<td>55</td>
<td>72122.44</td>
<td>595.8005</td>
</tr>
</tbody>
</table>

\[ 61 : \approx 34095 \text{ pb}^{-1} \]
\[ 63 : \approx 32858 \text{ pb}^{-1} \]

Total : \( \approx 526618 \text{ pb}^{-1} \)
The analysis uses kinematic fitting to improve resolution

**Kinematic Constraints:**
- **4-momentum:** $p_{\pi^+} + p_{\pi^-} + p_{\gamma\pi^0} + p_{\gamma\pi^0} + p_{\gamma\text{ISR}} = p_{e^+} + p_{e^-}$
- **Vertex:** $r_{\pi^+}^i + r_{\pi^-}^i + r_{\text{IPtube}}^i = r_{\text{IP}}^i$
  - $p^i = 4$-momentum
  - $r^i = 3$-position
  - IP = interaction point
  - IPtube = data derived constraint tube centered on IP
  - Total constraints = $4(p^i) + 2(r_{\pi^+}^i) + 2(r_{\pi^-}^i) + 2(r_{\text{IPtube}}^i) = 10$
  - *NO* $m_{\pi^0}$ CONSTRAINT!

- **To leading-order:** hadron system is back-to-back with $\lambda_{\text{ISR}}$
  - $\pi^0$ is reconstructed from $\gamma_{\pi^0} + \gamma_{\pi^0}$
**Reference: Pre Kinematic Fit Cuts**

Table 1: $\tau$-skim good track cuts. Cuts are in the lab frame.

<table>
<thead>
<tr>
<th>Cut Quantity</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse momentum</td>
<td>$p_t \geq 0.1 \text{ GeV}/c$</td>
</tr>
<tr>
<td>Transverse distance impact parameter</td>
<td>$</td>
</tr>
<tr>
<td>$z$ distance impact parameter</td>
<td>$</td>
</tr>
<tr>
<td>Polar angle impact parameter</td>
<td>$</td>
</tr>
<tr>
<td>Hits count</td>
<td>$nhits \geq 0$</td>
</tr>
</tbody>
</table>

Basic Tau Skim with good tracks

Table 2: Pre-kinematic fitting cuts.

<table>
<thead>
<tr>
<th>Cut Quantity</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good positive track count</td>
<td>$\hat{N}_{trk+} = 1$</td>
</tr>
<tr>
<td>Good negative track count</td>
<td>$\hat{N}_{trk-} = 1$</td>
</tr>
<tr>
<td>Good photon count</td>
<td>$N_\gamma \geq 3$</td>
</tr>
<tr>
<td>Good photon lab frame energy</td>
<td>one $\gamma$ with $E &gt; 2.75 \text{ GeV}$</td>
</tr>
</tbody>
</table>

Interesting candidate events
# Reference: Post Kinematic Fit Cuts

Table 3: The post-kinematic fitting cuts. There are 10 dof for the kinematic fit.

<table>
<thead>
<tr>
<th>Cut Quantity</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic fit $\chi^2$ for c.m. frame 4-momenta.</td>
<td>$\chi^2_{kfit} \leq 40$</td>
</tr>
<tr>
<td>PID</td>
<td>see Table 4</td>
</tr>
<tr>
<td>Photon energies</td>
<td>see Table 5</td>
</tr>
</tbody>
</table>

Table 4: The PID cuts. The $\chi^2_{\mu-ID}$ is used to check the quality of the $\mu$-ID.

<table>
<thead>
<tr>
<th>Cut Quantity</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$-identification</td>
<td>$e$-ID($\pi^+$) $\leq 0.1$ and $e$-ID($\pi^-$) $\leq 0.1$</td>
</tr>
<tr>
<td>$\mu$-identification</td>
<td>$(\chi^2_{\mu-ID}(\pi^+) \leq 0$ or $\mu$-ID($\pi^+$) $&lt; 0.1)$ and $(\chi^2_{\mu-ID}(\pi^-) \leq 0$ or $\mu$-ID($\pi^-$) $&lt; 0.1)$</td>
</tr>
<tr>
<td>$\pi$-signal to $e$-background likelihood ratio</td>
<td>$L_{\pi/e}(\pi^+) \geq 0.1$ and $L_{\pi/e}(\pi^-) \geq 0.1$</td>
</tr>
<tr>
<td>$\pi$-signal to $K$-background likelihood ratio</td>
<td>$L_{\pi/K}(\pi^+) \geq 0.6$ and $L_{\pi/K}(\pi^-) \geq 0.6$</td>
</tr>
<tr>
<td>$\pi$-signal to $p$-background likelihood ratio</td>
<td>$L_{\pi/p}(\pi^+) \geq 0.4$ and $L_{\pi/p}(\pi^-) \geq 0.4$</td>
</tr>
</tbody>
</table>

Table 5: The photon energy cuts. This analysis uses $m_{\pi^0} = 0.1349766$ GeV/c$^2$.

<table>
<thead>
<tr>
<th>Cut Quantity(ies)</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon lab frame energies</td>
<td>$E(\gamma) \geq 100$ MeV</td>
</tr>
<tr>
<td>Photon c.m. frame energies</td>
<td>$E(\gamma_{ISR}) &gt; E(\gamma_{\pi^0})$</td>
</tr>
<tr>
<td>Neutral particle invariant mass</td>
<td>$</td>
</tr>
<tr>
<td>ISR photon c.m. frame polar angle</td>
<td>$25^\circ \leq \theta(\gamma_{ISR}) \leq 155^\circ$</td>
</tr>
</tbody>
</table>
Comparison of MC and Data: 3

Charged Pion PID

Data (no bkg sub)

MC

(dozens more comparison plots exist and look good)
BaBar and BESIII obtain similar $2\pi$ distributions

$J/\psi \rightarrow \pi^+ \pi^- \pi^0$

$0.75 < m(\pi^+\pi^-\pi^0) < 0.82 \text{ GeV}/c^2$

$1.00 < m(\pi^+\pi^-\pi^0) < 1.04 \text{ GeV}/c^2$

$1.10 < m(\pi^+\pi^-\pi^0) < 1.40 \text{ GeV}/c^2$

$1.40 < m(\pi^+\pi^-\pi^0) < 1.80 \text{ GeV}/c^2$

Points → Data
Open Histogram → MC
Shaded Histogram → Calculated Background Contributions

Final* Belle $\sigma(e^+e^-\rightarrow\pi^+\pi^0\pi^0)$
(selected mass regions; linear scale)

*Systematic errors, background leakage, and small radiative correction checks to be completed in near future
$m(\pi^-\pi^0)$ vs. $m(\pi^+\pi^0)$ Dalitz plots

- **ω-region**: Data vs. Signal MC
- **φ-region**: Data vs. Signal MC

<table>
<thead>
<tr>
<th>$m^2(\pi^-\pi^0)$:Events [$/0.01$ [GeV$^2$/c$^4$]]</th>
<th>$m^2(\pi^+\pi^0)$:Events [$/0.01$ [GeV$^2$/c$^4$]]</th>
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<th>$m^2(\pi^-\pi^0)$:Events [$/0.025$ [GeV$^2$/c$^4$]]</th>
<th>$m^2(\pi^+\pi^0)$:Events [$/0.025$ [GeV$^2$/c$^4$]]</th>
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</table>
\( m(\pi^-\pi^0) \) vs. \( m(\pi^+\pi^0) \) Dalitz plots

\[ \omega' - \omega'' \text{-region} \]
\[ \text{Data} \]

\[ \omega' - \omega'' \text{-region} \]
\[ \text{Signal MC} \]

\[ J/\psi \text{-region} \]
\[ \text{Data} \]

\[ J/\psi \text{-region} \]
\[ \text{Signal MC} \]
$m(\pi^-\pi^0)$ vs. $m(\pi^+\pi^0)$ Dalitz plots

$\omega'\omega''$-region

Data

$\omega'\omega''$-region

Signal MC