Measurement of the lifetime of tau-lepton

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For the Belle Collaboration
Measurement of $\tau$-lepton lifetime, motivation

Precise measurement of the tau lifetime is necessary for the tests of lepton universality in the SM: $g_e = g_\mu = g_\tau$

\[
\Gamma(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma)) = \frac{\mathcal{B}(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma))}{\tau_L} = \frac{g^2_\ell g^2_e}{32M^2_W} \frac{m^5_L}{192\pi^3} F_{\text{corr}}(m_L, m_\ell) 
\]

\[
F_{\text{corr}}(m_L, m_\ell) = f(x) \left( 1 + \frac{3}{5} \frac{m^2_\ell}{M^2_W} \right) \left( 1 + \frac{\alpha(m_L)}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \right)
\]

\[
f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \quad x = m_\ell/m_L
\]

\[
\mathcal{B}(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)) = 1
\]

\[
\frac{g_\tau}{g_e} = \sqrt{\frac{\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\tau_\tau} \frac{m^5_\mu}{m^5_\tau} \frac{F_{\text{corr}}(m_\mu, m_\ell)}{F_{\text{corr}}(m_\tau, m_\mu)}}, \quad \frac{g_\tau}{g_e} = 1.0024 \pm 0.0021 \text{ (HFAG2012)}
\]

\[
\frac{g_\tau}{g_\mu} = \sqrt{\frac{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))}{\tau_\tau} \frac{m^5_\mu}{m^5_\tau} \frac{F_{\text{corr}}(m_\mu, m_\ell)}{F_{\text{corr}}(m_\tau, m_\mu)}}, \quad \frac{g_\tau}{g_\mu} = 1.0006 \pm 0.0021 \text{ (HFAG2012)}
\]
Measurement of $\tau$-lepton lifetime, motivation

S. Schael et al. [ALEPH, DELPHI, L3, OPAL, LEP EWG]

$$\frac{2B(W \rightarrow \tau \nu_\tau)}{B(W \rightarrow \mu \nu_\mu) + B(W \rightarrow e \nu_\theta)} = 1.066 \pm 0.025$$

2.6\sigma deviation from the Standard Model
Previous measurements

• Current PDG lifetime
  \[ \tau = (290.6 \pm 1.0) \times 10^{-15} \text{ sec} \]
  \[ ct = 87.11 \pm 0.30 \ \mu\text{m} \]
  Obtained at LEP experiments.

• BaBar result (at \( L = 80 \ \text{fb}^{-1} \), Tau’04 workshop)
  \[ \text{Nucl. Phys. B (Proc. Suppl.) 144 (2005) 105-112} \]
  \[ \tau = (289.40 \pm 0.91(\text{stat}) \pm 0.90(\text{syst})) \times 10^{-15} \text{ sec} \]

• BaBar analyzed topology 3-1. The analyzed variable was
  \[ \lambda_t = \left( x_p - x_d \right) \cdot \frac{\vec{p}_t}{\sin \theta} \]
Data and Monte Carlo samples

- **Data:** \( L = 711 \text{ fb}^{-1} \) (on- and off-resonance of \( \Upsilon(4S) \)).
- **Monte Carlo:**
  - Standard tau-tau sample prepared with KKMC generator with statistics equal to the luminosity of the Data.
  - We generated two additional \( e^+e^- \rightarrow \tau^+\tau^- \rightarrow 3\pi\gamma \ 3\pi\gamma \) samples with the life times \( c\tau = 84 \mu m \) and \( c\tau = 90 \mu m \) (about \( \pm 10\sigma \) (PDG) from the nominal value)
  - For the background estimation we used:
    - Standard EVTGEN light quarks, charm and beauty samples corresponding to the luminosity of the Data
    - gamma-gamma \( \rightarrow \) hadrons generated with PYTHIA
Measurement of $\tau$-lepton lifetime, method

- In the CM frame for the reaction $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow 3\pi\nu 3\pi\nu$ flight directions of $\tau^+$ and $\tau^-$ are back-to-back.
- Energy of each $\tau$-lepton is $\sqrt{s}/2$.
- Each $\tau$-lepton is decayed into $\tau \rightarrow 3\pi\nu$; mass of $\tau$-lepton is taken from PDG; neutrino mass assumed to be zero.

$$\cos\theta = \frac{2E_\tau E_x - m_\tau^2 - m_x^2}{2P_\tau P_x} = \frac{2E_\tau E_x - m_\tau^2 - m_x^2}{2\sqrt{(E_\tau^2 - m_\tau^2)P_x}}$$

- Two solutions of quadratic equation are possible $\tau$-lepton flight directions.
- $n_+^\tau$ is the unit vector in the direction of the positive $\tau$-lepton.
- $n_+^\tau = \sqrt{x^2 + y^2 + z^2}$
Angle between reconstructed and true $\tau$-direction for $\tau\tau$ Monte Carlo events.

Monte Carlo samples:
- Mean solution
  $\bar{n} = \frac{n_1 + n_2}{2}$
- $n_1$ - True solution
- $n_2$ - Wrong solution
- Thrust direction as $\tau$-direction
Obtaining the lifetime

• In laboratory frame we have two vertices and two momenta of $\tau$-leptons (two directions)

• This crossed-lines system has the following parameters:
  – $dl$ – distance between crossed-lines
  – $l_1 = (\vec{V}_1 - \vec{V}_{01}) \cdot \vec{n}_1$ – signed distance from the point of closest approach to the corresponding $\tau$-decay vertex in the direction of $\tau$-lepton momentum

• $c\tau_1 = \frac{l_1}{(\beta\gamma)_1}$

• From the analogous calculation $c\tau_2 = \frac{l_2}{(\beta\gamma)_2}$
Event selection

- The analyzed topology is 3-3 without $\pi^0$s.
  1. There are exactly 6 charged tracks compatible with the pion hypothesis with zero net charge.
  2. There are no $K^0_s$, $\Lambda$ and $\pi^0$.
  3. Thrust value (in CM frame) is greater than 0.9
  4. $P_t^2$ of the pion system is greater than 0.25 GeV$^2$
  5. $4 \text{ GeV} < m(6\pi) < 10.25 \text{ GeV}$
  6. Event is divided into two hemispheres by the plane perpendicular to the thrust axis. In each hemisphere there should be 3 pions with the net charge $\pm 1$.
  7. Pseudomass of each triplets of pions; $m_{\text{Min}}(3\pi) < 1.8 \text{ GeV}$
  8. Each triplet should be fitted to the vertex with $\chi^2 < 20$
  9. Discriminant for the system of equations $D>-0.05$
  10. Distance between crossed lines $dl < 0.02 \text{ cm}$
Discriminant distribution for solution of $\tau$-lepton direction

All above cuts are applied except the cut on $d_l$.

- **Data**

Monte Carlo samples:
- Evtgen-uds
- Evtgen-charm
- Evtgen-charged + mixed
- $\gamma\gamma \rightarrow$ hadrons

All Monte Carlo samples are normalized to the integrated luminosity of the Data.

We use events with $D > -0.05$, For negative $D$ we take $D=0$. 

$D$ – discriminant of quadratic equation
All above cuts are applied.

- Data

Monte Carlo samples:
- Evtgen-uds
- Evtgen-charm
- Evtgen-charged + mixed
- $\gamma\gamma \rightarrow$ hadrons

All Monte Carlo samples are normalized to the integrated luminosity of the Data.

We select events with distance smaller than 0.02 cm.
Dependence of the lifetimes of the selected taus on the applied cuts

The strongest dependence is on the last cut $dl<0.02cm$
Resolution function
Parameterization of the resolution function

\[ H(x) = P_1 \cdot R(x, P_2, ..., P_6) = \]

\[ = P_1 \cdot (1 - 2.5x) \cdot \exp \left[ -\frac{(x - P_2)^2}{2(P_3 + P_4|x - P_2|^{1/2} + P_5|x - P_2| + P_6|x - P_2|^{3/2})^2} \right] \]

\( P_1 - P_6 \) are free parameters
Resolution functions for different MC samples

- Standard MC (SVD1+SVD2) with \( \sigma = 87.11 \mu m 
- Standard MC (SVD1) with \( \sigma = 87.11 \mu m 
- Standard MC (SVD2) with \( \sigma = 87.11 \mu m 
- MC with \( \sigma = 84 \mu m 
- MC with \( \sigma = 90 \mu m 

\( c_\tau_{\text{reconstructed}} - c_\tau_{\text{true}} \) (cm)
Fitting function for $c\tau$ distributions in data and MC

$$F(x) = P_1 \int e^{-t/P_2} R((x - t), P_3, ..., P_7) dt + A_{uds} R(x, P_3, ..., P_7) + Bkg_{cb}(x)$$

7 free parameters $P_1 - P_7$

$A_{uds}$ - fixed parameter for contribution of the background from light quarks events.

c$\tau$ distributions for them is well described by resolution function $R(x, P_3, ..., P_7)$

Contribution from charm and beauty events $Bkg_{cb}$ was determined from MC
Result of the fit of the real data

Data distribution and fit
Light quarks from fit
Light quarks from MC
Charm and beauty from MC
MC correction of the fit parameter $P_2$

Dependence of the lifetime parameter $P_2$ obtained from the fit $(P_2-87) \text{ mkm}$ on the true input lifetime in the generator $c\tau-87 \text{ mkm}$.

After the MC correction of the parameter $P_2$ obtained from the fit of the data we get

\[
\langle c\tau \rangle = 86.99 \pm 0.16 \text{ (stat.) \mu m} \\
\langle \tau \rangle = 290.17 \pm 0.53 \text{ (stat.)} \cdot 10^{-15} \text{ s}
\]
Analysis of the systematics

1. Calibration of the alignment of the vertex detector
2. Asymmetry of the resolution function R
3. Choice of the range of the fit of reconstructed $c\tau$ distribution
4. Calibration of the beam energy
5. Accuracy of the description of ISR and FSR by MC
6. Accuracy of the estimation of the contributions of background events
7. Stability of the result with respect to the value of the last cut on $d_\ell$
8. Accuracy of the knowledge of the mass of $\tau$-lepton
9. We also checked the stability of the result for the different periods of the Belle operation and for different configurations of the tacking system
Uncertainty due to alignment of the vertex detector

- We generated five MC samples of $\tau^+\tau^-$ events which decay to $3\pi\nu3\pi\nu$ with the statistics of each sample $\sim$1.2 of the statistics of the data.
- For these events shifts of the DSSD plates were done randomly by 10 $\mu$m along X/Y/Z axis.
- For these events the random rotations of DSSD plates were done by angle 0.1 mrad.
- The maximal deviation of the parameter $P_2$ (distorted) from $P_2$ (without distortion) is 0.07 $\mu$m.
- For the same samples we performed the global SVD shifts and rotations with respect to drift chamber by 20 $\mu$m and 1 mrad respectively.
- The maximal deviation of the $P_2$ is 0.06 $\mu$m.
- For the estimation of the uncertainty we take the value $\sqrt{0.07^2 + 0.06^2} = 0.09$ $\mu$m.
Asymmetry of the resolution function and accuracy of ISR and FSR description by KKMC generator

• Monte Carlo predicts some asymmetry of the resolution function (factor $(1+2.5\cdot x)$ in the parameterization of $R$) with accuracy $2.5\pm0.2$

The result, obtained from the fit without this factor is different by $0.03 \mu m$. This value is taken as the systematics estimation

• The accuracy of the ISR and FSR description is checked by comparison of the data and MC events for the reaction $e^+e^- \rightarrow \mu^+\mu^- (n\gamma)$ reaction. Comparing the distributions $M(\mu^+\mu^-)-2\cdot E_{beam}$ we found the relative accuracy $2.1\cdot 10^{-4}$ in the $\tau$ lifetime
Accuracy of the estimation of background contribution

Variation of the background contribution in the fitting function. In particular performed the fits of the data distributions with uds contribution at the level of 50%, 100%, 150% and 200% of MC prediction. The variation of P2 parameter is within ±0.01 μm.
The stability of the result to the variation of the selection criteria, in particular to the cut on \( dl \)

The values of the fit parameter \( P2 \) in data and MC as function of the value of the cut on \( dl \)

MC corrected measured values of the \( \tau \) lifetime as function of the value of the cut on \( dl \)
# Systematics summary

<table>
<thead>
<tr>
<th>Source of Systematics</th>
<th>$\Delta(c\tau)$ in $\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVD alignment</td>
<td>0.090</td>
</tr>
<tr>
<td>Asymmetry of R-function</td>
<td>0.030</td>
</tr>
<tr>
<td>Fit range</td>
<td>0.020</td>
</tr>
<tr>
<td>ISR and FSR description</td>
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</tr>
<tr>
<td>Beam energy calibration</td>
<td>0.016</td>
</tr>
<tr>
<td>Background contribution</td>
<td>0.010</td>
</tr>
<tr>
<td>Error of the $\tau$-lepton mass</td>
<td>0.009</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.101</strong></td>
</tr>
</tbody>
</table>
Final result

\[ <c\tau> = 86.99 \pm 0.16 \text{ (stat.)} \pm 0.10 \text{ (syst.)} \mu m \]

\[ <\tau> = (290.17 \pm 0.53 \text{ (stat.)} \pm 0.33 \text{ (syst)}) \cdot 10^{-15} \text{ s} \]
Lifetime difference between $\tau^+$ and $\tau^-$

The difference of the $P_2$ parameters for $\tau^+$ and $\tau^-$ is $0.07 \pm 0.33 \, \mu m$

The systematics uncertainty is at least order of magnitude smaller than the statistical one

Upper limit is

$$|\tau(\tau^+) - \tau(\tau^-)| / \tau_{\text{average}} < 7.0 \cdot 10^{-3} \text{ at 90\% CL}$$

This measurement is done for the first time!
Comparison with the previous measurements

\[ \tau_t = 290.6 \pm 1.0 \quad \text{mean PDG} \]

- CLEO: \(289.0 \pm 2.8 \pm 4.0\)
- OPAL: \(289.2 \pm 1.7 \pm 1.2\)
- ALEPH: \(290.1 \pm 1.5 \pm 1.1\)
- L3: \(293.2 \pm 2.0 \pm 1.5\)
- DELPHI: \(290.9 \pm 1.4 \pm 1.0\)
- Belle: \(290.17 \pm 0.53 \pm 0.33\)
Lepton universality test

\[ \tau_\tau = \tau_\mu \left( \frac{g_\mu}{g_\tau} \right)^2 \left( \frac{m_\mu}{m_\tau} \right)^5 \text{Br}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \frac{f \left( \frac{m_\tau^2}{m_\mu^2} \right) F_\mu F_\mu^{rad}}{f \left( \frac{m_\tau^2}{m_\mu^2} \right) F_\tau F_\tau^{rad}} \]

Belle result:

\[ \left( \frac{g_\tau}{g_\mu} \right)^2 = 1.0041 \pm 0.0035 \]
Prospects for Belle II

• In the present result the dominant uncertainty is statistical one

• For the statistics ~50 times of Belle I we expect the dominance of systematics uncertainty. From the present overall uncertainty of $\tau \sigma_{0.2}$ μm we will have overall accuracy $0.1$ μm
Prospects for LHCb

For the present integrated luminosity 3fb$^{-1}$ the estimated statistics of $\tau$-leptons is about $120\cdot10^9$ in LHCb acceptance. This should be compared with $2\cdot10^9 \tau$-leptons at Belle or $100\cdot10^9$ at Belle II. LHCb has big potential for the search of LFV decays of $\tau$-leptons such as $\tau \rightarrow 3\mu$.

On the other hand for the lifetime measurement almost all LHCb $\tau$’s are not useful because they are the products of the decays of Ds-mesons and b-hadrons. For the lifetime measurement only prompt $\tau$’s are useful.

The other problem for LHCb is the knowledge of the energy spectrum of $\tau$-leptons which is needed for the evaluation of mean kinematic factor $\beta\gamma$.

Our conclusion from the all above: for the $\tau$-lepton lifetime measurement the LHCb is not competitive with the $e^+e^-$ B-factories.
Conclusions

• Belle performed the measurement of \( \tau \)-lepton lifetime with the accuracy 1.6 times better than the present PDG value:
  \[ \tau = (290.17 \pm 0.62) \cdot 10^{-15} \text{s} \]

• For the first time the upper limit on the lifetime difference between \( \tau^+ \) and \( \tau^- \) was obtained:
  \[ \frac{|\tau(\tau^+) - \tau(\tau^-)|}{\tau_{\text{average}}} < 7.0 \cdot 10^{-3} \text{ at 90% CL} \]

• At Belle II we expect the improvement of the \( \tau \) lifetime accuracy by factor \( \sim 2 \)